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High Gravitational Waveform Accuracy at Null Infinity

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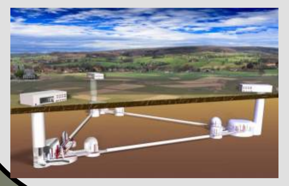
Motivation

- Gravitational wave observatories like LIGO, Virgo, or ET will detect any vibration, not only gravitational waves.
- Accurate waveforms are required to complement their sensitivity and enable scientific interpretation
- Perturbative wave extraction at finite radius have errors due to: gauge effects, near fields, back reaction.
- Cauchy-characteristic extraction (CCE): extends the simulation to future positive null Infinity where the waveform is computed in inertial coordinates.




Advanced Accuracy Standards Criteria

Characteristic evolution is embedded in the Cauchy evolution.
Two sets of initial boundary data:

1. Cauchy on the inner world-tube: close quasicircular black-hole binary with orbital frequency MΩ = 0.05
2. Characteristic for the initial null hyper surface in order to suppress the incoming radiation, provide continuity Cauchy data at the extraction worldtube and vanish at plus null infinity.

$$J = \frac{J|_{x_E}(1-x)x_E}{(1-x_E)x}$$

- The characteristic data is reconstructed from a spectral decomposition of Cauchy data between two world-tubes.
- The data is used to calculated the Bondi-Sachs metric functions on the world-tube in characteristic coordinates

$$ds^2 = -\left(e^{2\beta}\frac{V}{r} - r^2 h_{AB}U^A U^B\right)du^2 - 2e^{2\beta}dudr - 2r^2 h_{AB}U^B dudx^A + r^2 h_{AB}dx^A dx^B$$

Cauchy-Characteristic Extraction

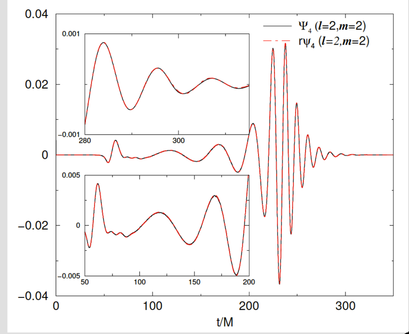
The characteristic extracted waveform is presented in terms of the Bondi News N in inertial Bondi coordinates the or the Weyl tensor Ψ

$$\Psi = \partial_u N \qquad \Psi_4 = -2\bar{\Psi} \qquad \Psi_4^0(u, x^A) = \lim_{r\to\infty} r\psi_4$$

The perturbative waveform is extrapolated to infinity with formula:

$$\lim_{R\to\infty}[R\psi_4^{lm}(R,t)] = r\psi_4^{lm}(r,t) - \frac{(l-1)(l+2)}{2}\int_0^t dt\psi_4^{lm}(r,\tau)d\tau + \mathcal{O}(r^2)$$

Perturbative extrapolation is essential to obtain the phase agreement shown from the early stages and throughout the final ringdown between the Characteristic and Cauchy waveforms for the (2,2) mode.



Waveforms

Translation from the frequency domain into the time domain: the error of a numerical waveform with strain component h(t) is:

$$\mathcal{E}_0 = \frac{||\delta h||}{||h||}$$

Correspondence given by the signal to noise ratio: $\rho^2 = \int_0^\infty \frac{4|\hat{h}(f)|^2}{S_n(f)}df$

1. Accuracy for detection: $\mathcal{E}_k \leq C_k\frac{\eta_c}{\rho}$
2. Accuracy for measurement: $\mathcal{E}_k \leq C_k\sqrt{2\epsilon_{max}}$

- Loss due to: template mismatch ε_{max} and detector calibration η_c

Required accuracy or detection:

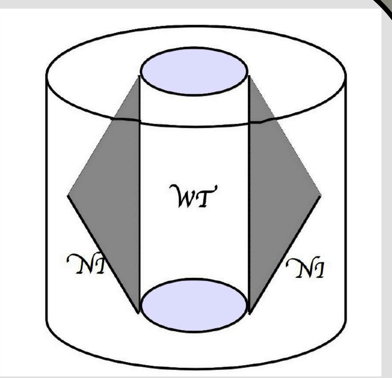
- e_{max} = 0.005, 0.24 ≤ C_1 ≤ 0.8=>ε_1 ≤ 0.1C_1 ≤ 0.024

Required accuracy for measurement:

- h_{min} = 0.4, C_1 =0.24, r=100 =>ε_1 ≤9.6 ×10^{-2}/r ≤9.6×10^{-4}

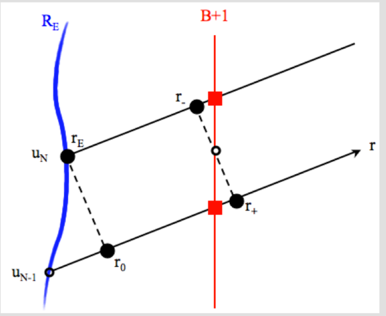
Convergence Limitations

The extraction worldtube with fixed Cartesian radius moves with respect to the null grid.



The change in location of the boundary points introduces a stochastic grid-dependent source of error at the worldtube boundary.

This random separation enters into the 2nd order accurate approximations in the start-up algorithm altering the convergence.



The waveforms are only first order accurate as a result of the asymptotic limits at Infinity+.

Run Time 2nd –order Convergence

Evolution variables

Variable	$Rate_{Re}$	$Rate_{Im}$
β	2.01	2.01
J	2.23	2.01
$J_{,x}$	2.03	2.33
Q	2.02	2.04
U	1.99	1.96
W	1.97	2.00

Asymptotic quantities

Variable	$Rate_{Re}$	$Rate_{Im}$
β	2.01	2.01
J	1.80	2.18
$J_{,x}$	1.23	1.20
Q	1.33	1.19
U	1.99	1.96
W	1.56	1.50

Bondi News and Weyl component, near the peak of the signal, for R=20M

Variable	$Rate_{Re}$	$Rate_{Im}$
N	1.59	1.56
$\partial_u N$	1.57	1.55
Ψ	1.16	1.14

Richardson Extrapolation

- We use results from three different resolutions:

$$F_1 = f(\Delta),\ F_2 = F(2\Delta)\ F_4 = F(4\Delta)$$

- We construct the 2nd order accurate functions

$$F_I = 2F_1 - F_2 \qquad F_{II} = 2F_2 - F_4$$

- We extrapolate to obtain 3rd order accurate function.

$$F_E = \frac{8}{3}F_1 - 2F_2 + \frac{1}{3}F_4$$

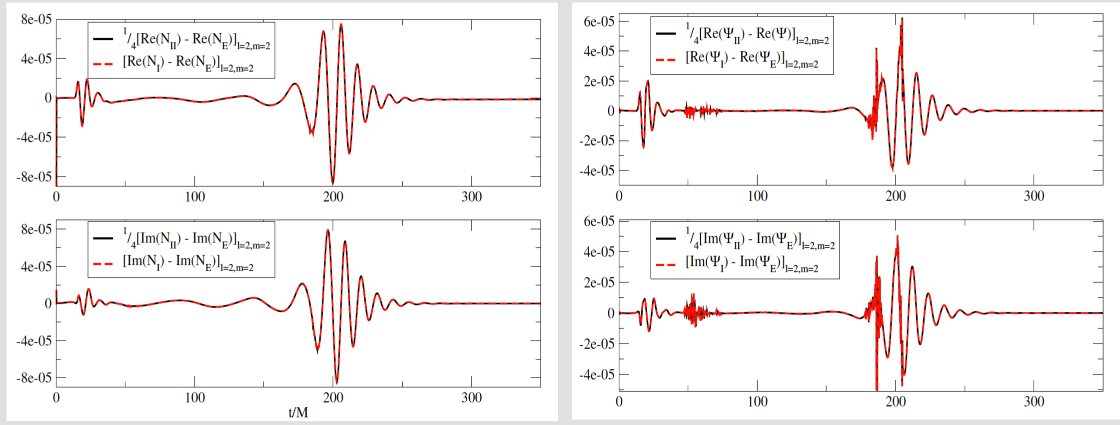
- We estimate the absolute error in the waveform:

$$\delta\Psi = \Psi_I - \Psi_E \qquad \delta\psi_4 = (\frac{1}{2}r\psi_4 + \bar{\Psi}) \qquad \delta N = N_I - N_E$$

Postprocess 2nd–order Convergence

The figure reveals two sources of extraneous "junk" radiation:

1. From the mismatch between the initial characteristic and Cauchy data, visible at early times especially for RE = 20M.
2. From the choice of comformally flat initial Cauchy data,for all three extraction radii and accounts for the early double hump.



Requirements for the Bondi News

1. The criterion for detection <u>is satisfied</u> *throughout the entire binary mass range.* In addition, the detection criterion is unaffected by choice of extraction radius.
2. The criterion for measurement <u>is also satisfied</u> *throughout the entire binary mass range.*

An optimal choice of extraction radius could minimize the errors

Variable	Re	Im
$\mathcal{E}_1(N)_{R=20}$	8.76×10^{-4}	8.74×10^{-4}
$\mathcal{E}_1(N)_{R=50}$	2.62×10^{-4}	2.60×10^{-4}
$\mathcal{E}_1(N)_{R=100}$	1.21×10^{-4}	1.22×10^{-4}
$\mathcal{E}_1(N_{\Delta R(20,100)})$	5.41×10^{-3}	5.55×10^{-3}
$\mathcal{E}_1(N_{\Delta R(50,100)})$	4.28×10^{-3}	4.51×10^{-3}

Requirements for the Weyl Tensor

1. The criterion for detection <u>is satisfied</u> *throughout the entire binary mass range* in the high mass limit. In addition, the truncation error decreases with extraction radius.
2. The values at all three extraction radii satisfy the measurement requirement for advanced LIGO signal-to-noise ratio ρ = 100.

The importance of an optimal choice of extraction radius is stressed.

Variable	Re	Im
$\mathcal{E}_2(\Psi)_{R=20}$	1.138×10^{-3}	1.174×10^{-3}
$\mathcal{E}_2(\Psi)_{R=50}$	4.038×10^{-4}	3.531×10^{-4}
$\mathcal{E}_2(\Psi)_{R=100}$	2.810×10^{-4}	2.093×10^{-4}
$\mathcal{E}_2(\Psi_{\Delta R(20,100)})$	5.391×10^{-2}	4.148×10^{-2}
$\mathcal{E}_2(\Psi_{\Delta R(50,100)})$	1.937×10^{-2}	1.905×10^{-2}

Cauchy and Characteristic Differences

- The difference between characteristic and pertubative extraction is fairly independent of the extracted radii
- In the high mass limit these errors impact the measurement criterion only for signal to noise ratios ρ > 59 but they could be expected to be more significant for low mass binaries.
- Whether the error can be attributed to characteristic extraction or to perturbative extraction deserves further investigation.

Variable	Re	Im
$\mathcal{E}_2(\delta\psi)_{R=50}$	5.09×10^{-3}	5.08×10^{-3}
$\mathcal{E}_2(\delta\psi)_{R=100}$	6.81×10^{-3}	6.32×10^{-3}
$\mathcal{E}_2(\psi_{4,\Delta R(50,100)})$	3.13×10^{-2}	3.14×10^{-2}

Conclusions

- The aim of CCE is to provide a standardized waveform extraction tool for the numerical relativity community.
- The new extraction tool contains major improvements and corrections to previous versions and displays convergence.
- The error introduced by CCE satisfies the time domain criteria required for advanced LIGO data analysis.
- The importance of accurate waveforms to the gravitational wave astronomy has created an urgency for tools like CCE.
- The source code has been released to the public and is available as part of the Einstein Toolkit.
- We welcome applications to a variety of generic Cauchy codes implementing Einstein Equations of General Relativity.