Fall 10-3-2014

Characteristics of gravitational and electromagnetic radiation

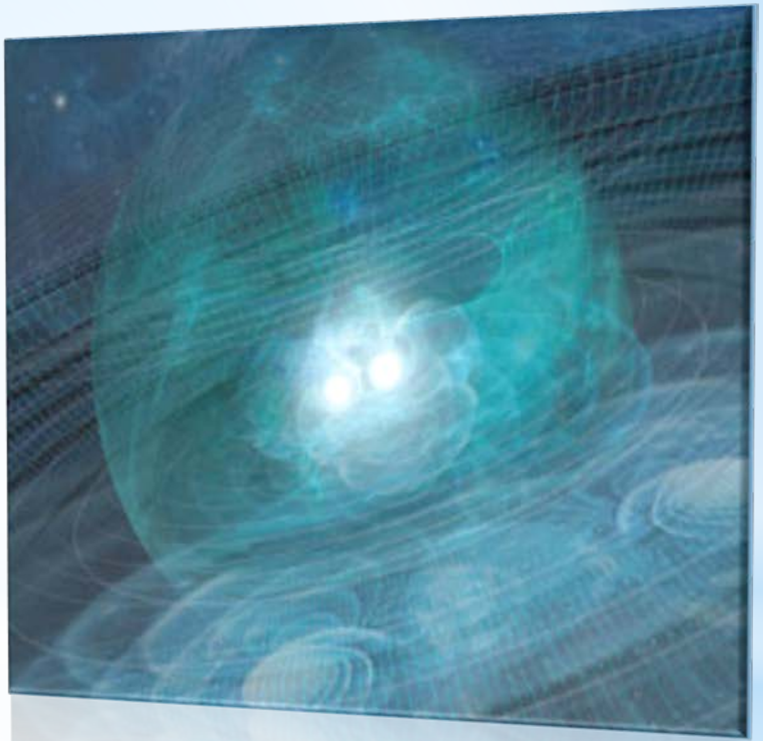
Maria Babiuc-Hamilton
Marshall University, babiuc@marshall.edu

* Characteristics of Gravitational and Electromagnetic Radiation

Maria Babiuc-Hamilton, Marshall University, Huntington, WV

MAS-APS Physics Meeting 2014, Oct 3-4, State College, PA

* The tale of the universe
    - Colliding black holes or even entire galaxies
    - The birth of a black hole in a supernova explosion
    - The beginning and growth pains of our universe
    - The ultimate question of what is space and time.

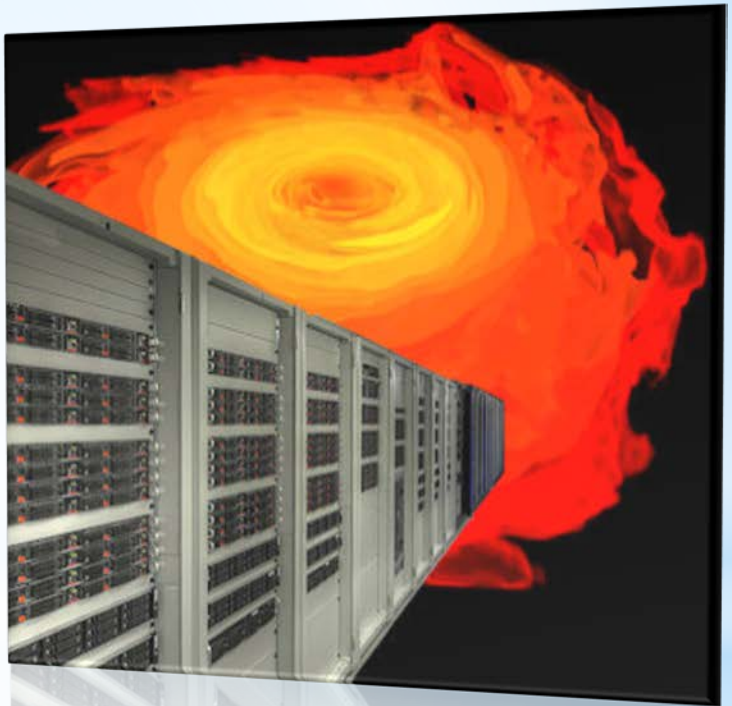


A New Astronomy

*Reality Check 1

* *Detection challenges
  * The strain is extremely small of magnitude: 10^{-3} the width of a proton
  * The detectors are swamped in noise
  * The need for accurate waveform templates

* Computational challenges
  - Numerical algorithms, approximations, and supercomputers are needed to simulate Einstein equations.
  - Boundary conditions and truncations are imposed.



*Reality Check 2

* Bondi (1962) proved mathematically the existence of gravitational waves at null infinity.
* He found an exact solution of Einstein equations:

$$ds^2 = -e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}du\,dr$$

$$+r^2 h_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

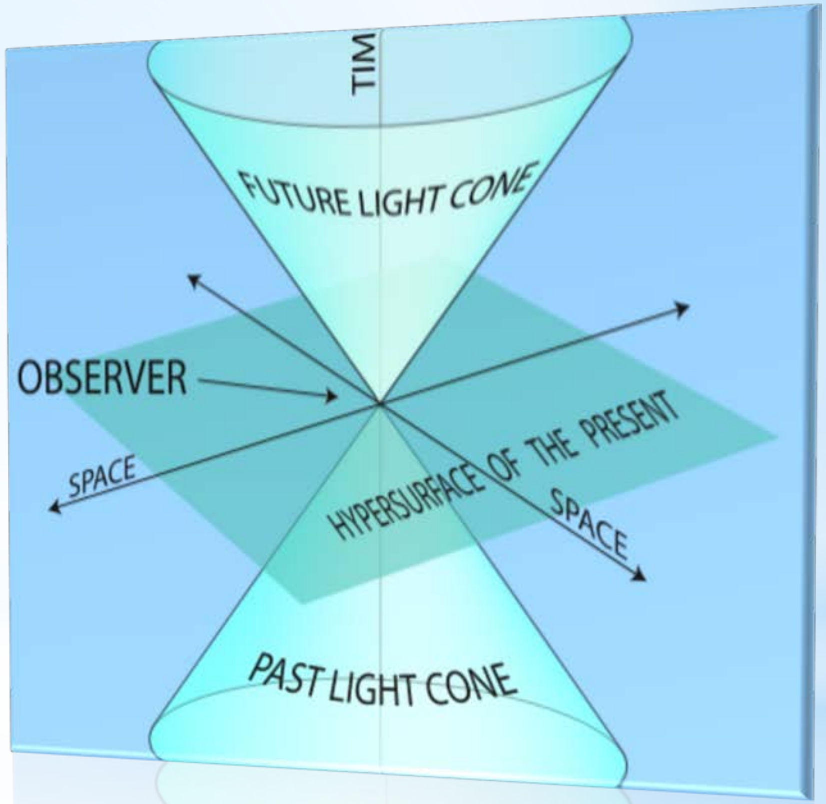
* Within this metric, he calculates the **loss of mass** due to the emission of gravitational waves

$$N = N_+ + iN_\times = \partial_t h_+ + \partial_t h_\times$$

*Bondi Makes the News

*The mass of a system is constant if and only if there is no **news**.*
*If there is **news**, the mass decreases as long as there are news.*

* Light rays are ***principal null directions*** in space-time for both gravitational and electromagnetic radiation.
* They are ***characteristic surfaces*** of both Einstein and Maxwell field equations.
* In characteristic coordinates, the equations split into time evolution and hypersurface integration equations.



*How Radiation Travels

*Gravitation and Light

* We consider the coupled Einstein-Maxwell system, together with Maxwell equation, on a Bondi metric.
* A null gauge field splits the coupled field equations into hypersurface, evolution and supplementary conditions.

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

$$T_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha\gamma} F^{\gamma}_{\ \beta} - \frac{1}{4} g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}\right)$$

$$F_{\alpha\gamma} = 2D_{[\alpha} A_{\beta]}; \; A^{u} = A_{r} = 0$$

$$D_{\beta} F^{\alpha\beta} - 8\pi J^{\alpha} = 0$$

*Main Equations for Light

1. With A_B initial, we get A_u by integrating J^u along the ray.
2. With A_B and A_u we get $\delta_u A_B$ by integrating J^B
3. With $\delta_u A_B$ known, we can obtain A_B on the next characteristic.

$$
\begin{aligned}
4\pi J^u = &-\frac{1}{r^2 e^{4\beta}} D_B\left(h^{BC}\partial_r A_C\right) \\
&+\frac{1}{e^{4\beta}}\left[\partial_r\left(\partial_r A_u + U^C\partial_r A_C\right) + \frac{2}{r}\left(\partial_r A_u + U^C\partial_r A_C\right)\left(1 - r\partial_r\beta\right)\right] \\
4\pi J^B = &\frac{1}{r^2 e^{2\beta}}\left[\partial_u\left(h^{BC}\partial_r A_C\right) + \partial_r\left(h^{BC}(2\partial_{[u}A_{C]} - \frac{V}{r}\partial_r A_C)\right)\right] \\
&+\frac{2}{r^2 e^{2\beta}}\left[\partial_r\left(U^D h^{BC}\partial_{[D}A_{C]}\right) + D_C\left(U^{[B}h^{C]D}\partial_r A_D\right)\right] \\
&+\frac{1}{e^{4\beta}}\left[\frac{2U^B}{r}\left(\partial_r A_u + U^C\partial_r A_C\right)\left(1 - r\partial_r\beta\right) + \partial_r\left(U^B(\partial_r A_u + U^C\partial_r A_C)\right)\right] \\
&+\frac{1}{r^4}\left[2D_C\left(h^{BE}h^{CD}\partial_{[E}A_{D]}\right) + 4\partial_C\beta\left(h^{BE}h^{CD}\partial_{[E}A_{D]}\right)\right]
\end{aligned}
$$

The “Memory” Effect

- The equation for J^r is a suplementary condition.
- It provides the information on the electric and magnetic parts of the null radiation “memory” effect: change in relative separation of two test particles.

$$4\pi J^r = -\frac{1}{e^{4\beta}}\left[D_B\left(U^B(\partial_r A_u + U^C\partial_r A_C)\right)\right]$$

$$-\frac{1}{e^{4\beta}}\left[\partial_u\left(\partial_r A_u + U^C\partial_r A_C\right) + 2\left(\partial_u\beta + \partial_C\beta\right)\left(\partial_r A_u + U^C\partial_r A_C\right)\right]$$

$$-\frac{1}{r^2 e^{2\beta}}\left[D_B\left(h^{BC}(2\partial_{[u}A_{C]} - \frac{V}{r}\partial_r A_C)\right) + 2D_B\left(h^{BD}U^C\partial_{[C}A_{D]}\right)\right]$$

$$D_B\left(h^{BC}E_C\right) + D_B\left(h^{BC}(\frac{V}{r}B_C - U^C B_r)\right) + \frac{1}{e^{2\beta}}D_B\left(U^B(E_r - U^C B_C)\right)$$

$$= 4\pi r^2 e^{2\beta} J^r + \frac{r^2}{e^{2\beta}}\left[\partial_u\left(U^C B_C - E_r\right) + 2\left(\partial_u\beta + \partial_C\beta\right)\left(U^C B_C - E_r\right)\right]$$

The Maxwell Tensor

- The terms T_{rr} , T_{rB} and T_{BC} are added to the main Einstein equations and integrated on hypersurface
- The potential $(A_u, A_B, \delta_u A_B)$ is provided by Maxwell equations.

$$T_{rr} = -\frac{h^{BC}}{4\pi r^2}\partial_r A_B \partial_r A_C$$

$$T_{rB} = -\frac{1}{4\pi e^{2\beta}}\partial_r A_B\left(\partial_r A_u + U^C \partial_r A_c\right) - \frac{2h^{CD}}{r^2}\partial_r A_C \partial_{[B} A_{D]}$$

$$T_{BC} = -\frac{1}{\pi e^{2\beta}}\partial_r A_C \partial_{[B} A_{u]} - \frac{U^D}{\pi e^{2\beta}}\partial_r A_C \partial_{[B} A_{D]}$$

$$-\frac{1}{4\pi e^{2\beta}}\frac{V}{r}\partial_r A_B \partial_r A_C - \frac{h^{DE}}{\pi r^2}\partial_{[B} A_{D]}\partial_{[C} A_{E]} - \ldots$$

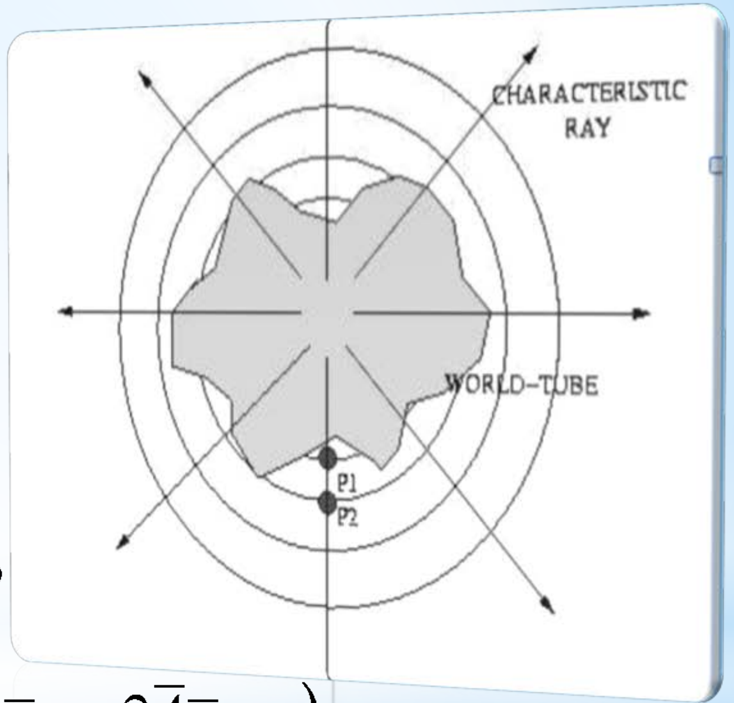
* Express variables in terms of a complex dyad (q_A, q_B)
* The unit metric is $q_{BC} = q_{(B}q_{C)}$

$$h_{BC} = \frac{J}{2}\bar{q}_B\bar{q}_C + \frac{\bar{J}}{2}q_B q_C + \frac{K}{2}q_{(B}\bar{q}_{C)}$$

$$\partial_C A_u = \frac{1}{2}\left(\eth A_u \bar{q}_C + \bar{\eth} A_u q_C\right);\ A = q^D A_D$$

$$D_C A_B = \frac{1}{4}\left(\eth A \bar{q}_C \bar{q}_B + \bar{\eth}\bar{A} q_C q_B + \bar{\eth} A q_C \bar{q}_B + \eth \bar{A} \bar{q}_C q_B\right)$$

$$-\frac{1}{2}h^{DE}\left(\nabla_C h_{BE} + \nabla_B h_{CE} - \nabla_E h_{CB}\right)A_D$$

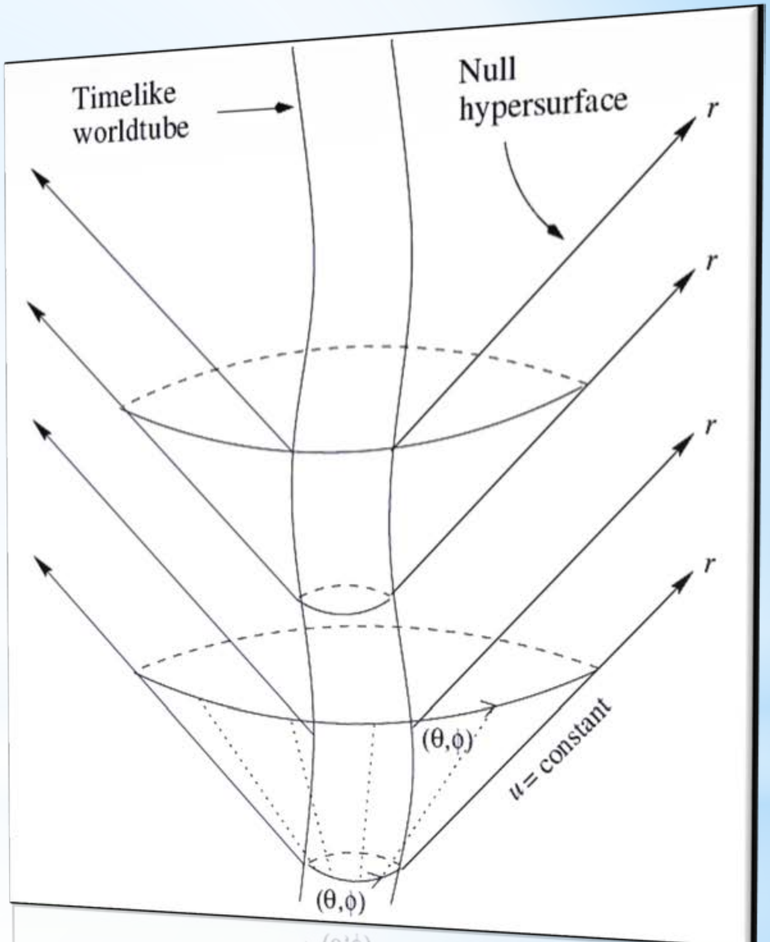


*The Hypersurface

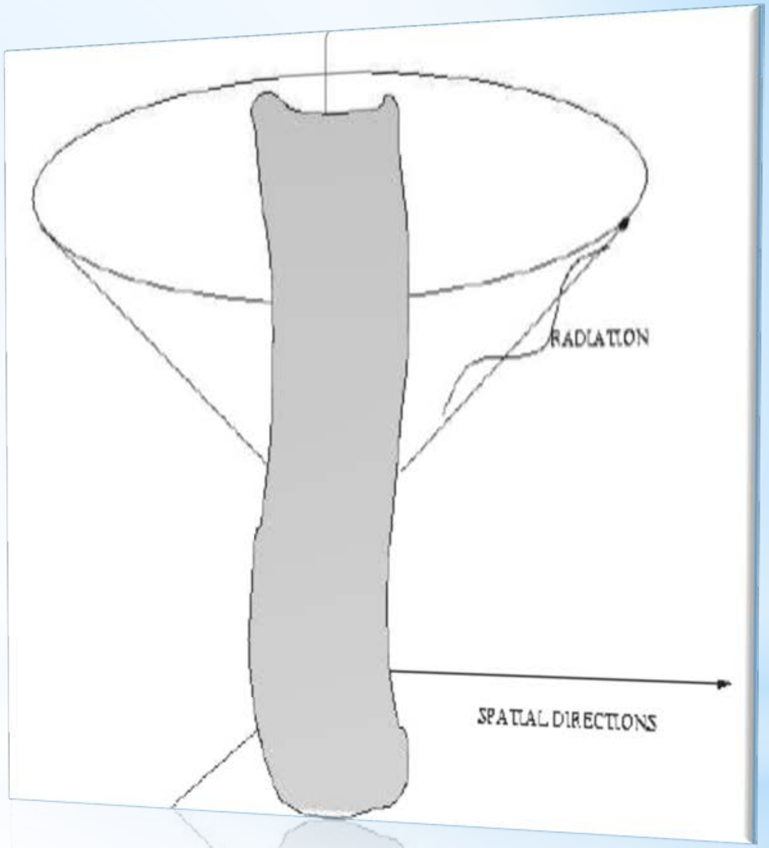
Characteristic Foliation

- Foliation of space & time by null rays ***u = t-r = const.***
- The *messy physics* happens within a timelike worldtube.
- Spherical 2D cones $(\theta, \varphi) = x^A$ are front-waves of the rays
- The null grid $(u,\ r,\ x^A)$ starts on the worldtube and transports the wave to positive null infinity.
- Penrose compactification:

$$x = \frac{r}{R+r},\ r \to \infty \Rightarrow x \to 1$$



* Einstein equations are evolved along the light rays, by ***marching*** on the outgoing characteristics.
* *Errors at machine precision*
* The gravitational waveforms are computed at positive null infinity on inertial Bondi coordinates in terms of the ***Bondi News*** and **Weyl** Ψ_4 scalar.

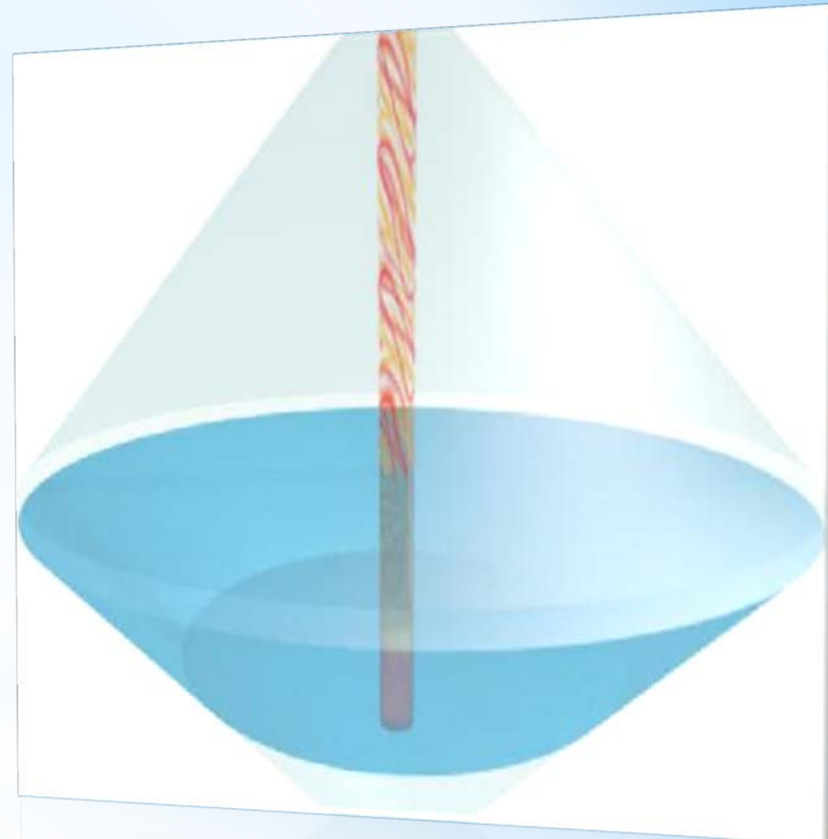


*The Characteristic Code

* Next:

1. Code the coupled Einstein-Maxwell equations
2. Implement a characteristic code for the Maxwell field

* Electromagnetic counterparts shed new light on the sources.
* Other interesting phenomena:
    - Gravitational and electromagnetic memory
    - Formation of trapped surfaces and horizons



*Adding to the News