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An analysis of the gravitational waves null memory

Maria Babiuc-Hamilton
Marshall University, babiuc@marshall.edu

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Maria Babiuc Hamilton, Marshall University, Huntington, WV

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*What is Memory

*Memory involves a process of physical changes in the structure of **receptors.**

- Short-term or temporary
- Long-term or permanent

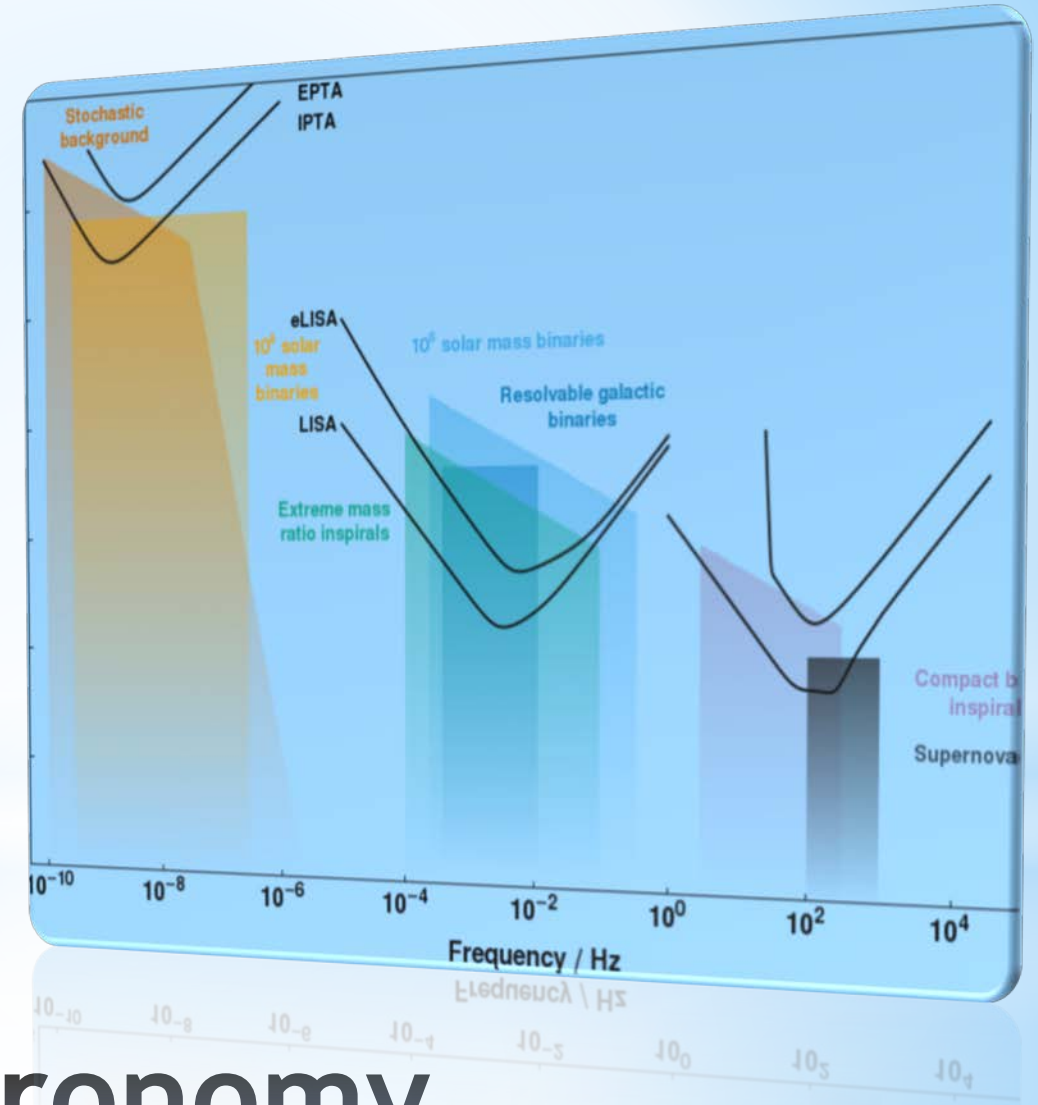
*Gravitational Waves

*__Ripples__ in the fabric of spacetime generated by:

- Colliding black holes or even entire galaxies
- The birth of a black hole in a supernova explosion
- The beginning and growth pains of our universe

A New Astronomy

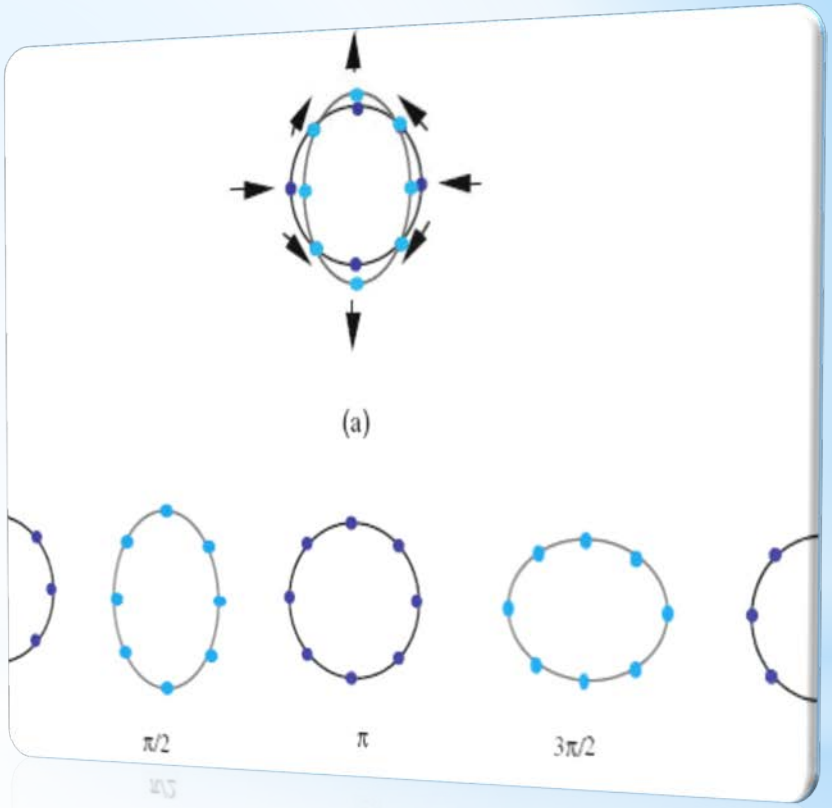
- Normal Astronomy sees with "light waves"
- **Gravitational Wave Astronomy** "sees" (*hears*) with gravitational waves
- The strain is extremely small: 10^{-3} the width of a proton



*Gravitational Wave Memory

*The "**memory**" of a **gravitational-wave** burst is the **permanent** relative displacement of receptors.

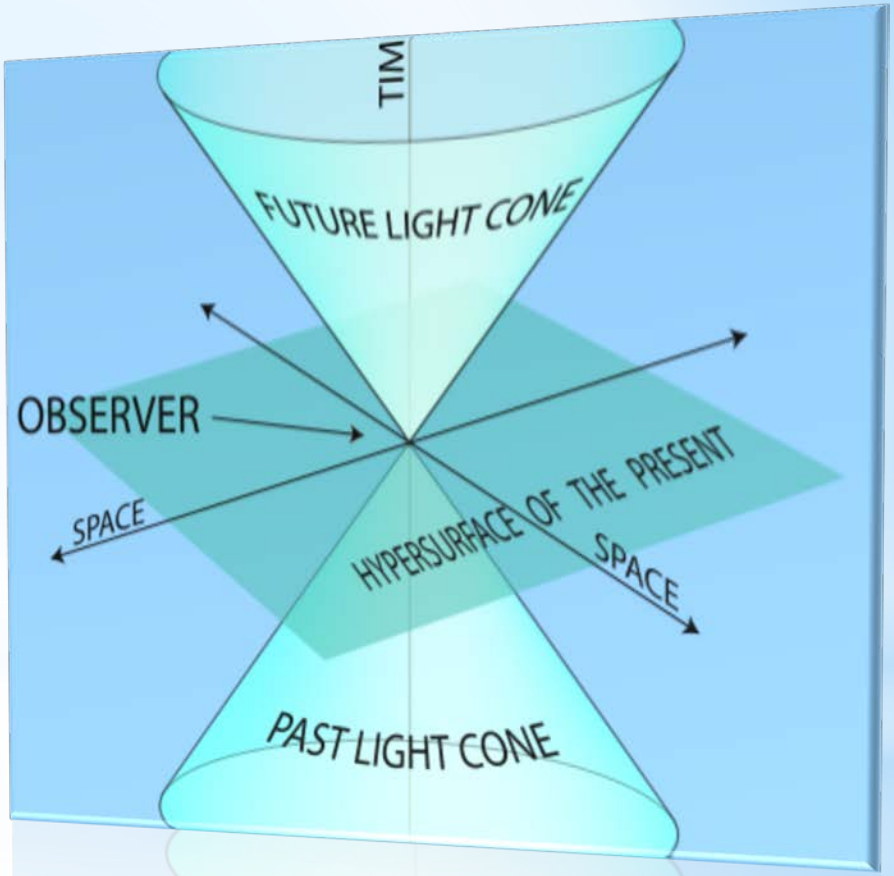
- Linear or "ordinary", due to anisotropic source emission
- Nonlinear or "null", produced by the gravitational waves propagating to null infinity



*How Radiation Travels

*Light rays are ***principal null directions*** in space-time for both gravitational and electromagnetic radiation.

*They are ***characteristic surfaces*** of both Einstein and Maxwell field equations.



*Gravitation and Light

- We consider the coupled **Einstein-Maxwell system of** equations, on a null space-time metric.
- A null gauge field splits the coupled field equations into spatial components, evolution equations, and conservation laws.

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

$$T_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha\gamma} F^{\gamma}_{\ \beta} - \frac{1}{4} g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}\right)$$

$$F_{\alpha\gamma} = 2D_{[\alpha} A_{\beta]}; \; A^{u} = A_{r} = 0$$

$$D_{\beta} F^{\alpha\beta} - 8\pi J^{\alpha} = 0$$

* Bondi (1962) proved mathematically the existence of gravitational waves at null infinity.
* He found an exact solution of Einstein equations:

$$ds^2 = -e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}dudr$$

$$+r^2 h_{AB}(dx^A - U^A du)(dx^B - U^B du)$$

* Within this metric, he calculates the **loss of mass** due to the emission of gravitational waves

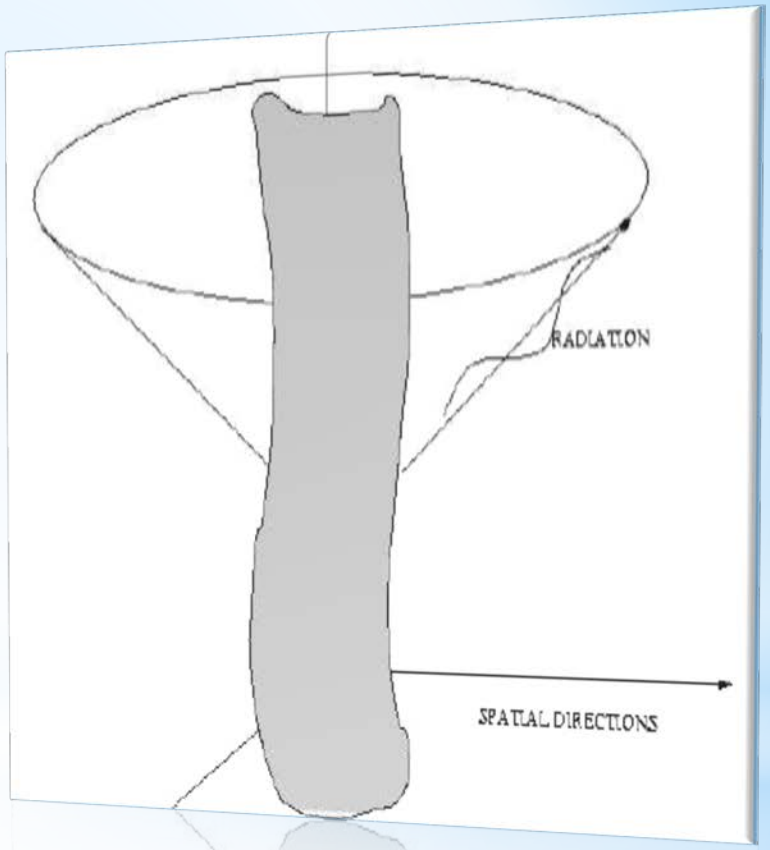
$$N = N_+ + iN_\times = \partial_t h_+ + \partial_t h_\times$$

* Bondi Makes the News

*The mass of a system is constant if and only if there is no **news**.*
*If there is **news**, the mass decreases as long as there are news.*

The Characteristic Code

- The equations are evolved along the light rays, by ***marching*** on the outgoing characteristics.
- *The messy physics is confined by the worldtube*
- The gravitational waveforms are computed at positive null infinity on inertial Bondi coordinates in terms of the ***Bondi News*** and **Weyl** Ψ_4 scalar.



1. J^u is a spatial equation that can be integrated radially.
2. J^B is an evolution equation that can be integrated in time.

$$4\pi J^u = -\frac{1}{r^2 e^{4\beta}} D_B\left(h^{BC}\partial_r A_C\right)$$
$$+\frac{1}{e^{4\beta}}\left[\partial_r\left(\partial_r A_u + U^C\partial_r A_C\right)+\frac{2}{r}\left(\partial_r A_u + U^C\partial_r A_C\right)\left(1-r\partial_r\beta\right)\right]$$
$$4\pi J^B = \frac{1}{r^2 e^{2\beta}}\left[\partial_u\left(h^{BC}\partial_r A_C\right)+\partial_r\left(h^{BC}(2\partial_{[u}A_{C]}-\frac{V}{r}\partial_r A_C)\right)\right]$$
$$+\frac{2}{r^2 e^{2\beta}}\left[\partial_r\left(U^D h^{BC}\partial_{[D}A_{C]}\right)+D_C\left(U^{[B}h^{C]D}\partial_r A_D\right)\right]$$
$$+\frac{1}{e^{4\beta}}\left[\frac{2U^B}{r}\left(\partial_r A_u + U^C\partial_r A_C\right)\left(1-r\partial_r\beta\right)+\partial_r\left(U^B(\partial_r A_u + U^C\partial_r A_C)\right)\right]$$
$$+\frac{1}{r^4}\left[2D_C\left(h^{BE}h^{CD}\partial_{[E}A_{D]}\right)+4\partial_C\beta\left(h^{BE}h^{CD}\partial_{[E}A_{D]}\right)\right]$$

*Main Equations for Light

*The “Memory” Effect

- *J^r gives a conservation law.
- *It provides the information on the electric and magnetic parts of the null radiation “memory” effect: change in relative separation of two test particles.

$$4\pi J^r = -\frac{1}{e^{4\beta}}\left[D_B\left(U^B(\partial_r A_u + U^C\partial_r A_C)\right)\right]$$

$$-\frac{1}{e^{4\beta}}\left[\partial_u\left(\partial_r A_u + U^C\partial_r A_C\right) + 2\left(\partial_u\beta + \partial_C\beta\right)\left(\partial_r A_u + U^C\partial_r A_C\right)\right]$$

$$-\frac{1}{r^2 e^{2\beta}}\left[D_B\left(h^{BC}(2\partial_{[u}A_{C]} - \frac{V}{r}\partial_r A_C)\right) + 2D_B\left(h^{BD}U^C\partial_{[C}A_{D]}\right)\right]$$

$$D_B\left(h^{BC}E_C\right) + D_B\left(h^{BC}(\frac{V}{r}B_C - U^C B_r)\right) + \frac{1}{e^{2\beta}}D_B\left(U^B(E_r - U^C B_C)\right)$$

$$= 4\pi r^2 e^{2\beta}J^r + \frac{r^2}{e^{2\beta}}\left[\partial_u\left(U^C B_C - E_r\right) + 2\left(\partial_u\beta + \partial_C\beta\right)\left(U^C B_C - E_r\right)\right]$$

*Adding to the News

*Next:

1. Code the coupled Einstein-Maxwell equations
2. Implement a characteristic code for the Maxwell field

*Electromagnetic counterparts shed new light on the sources.

*Other interesting phenomena:

- Gravitational and electromagnetic memory
- Formation of trapped surfaces and horizons