Spring 5-16-2014

Gravity & Electromagnetism on the Null Cone

Maria Babiuc-Hamilton
Marshall University, babiuc@marshall.edu

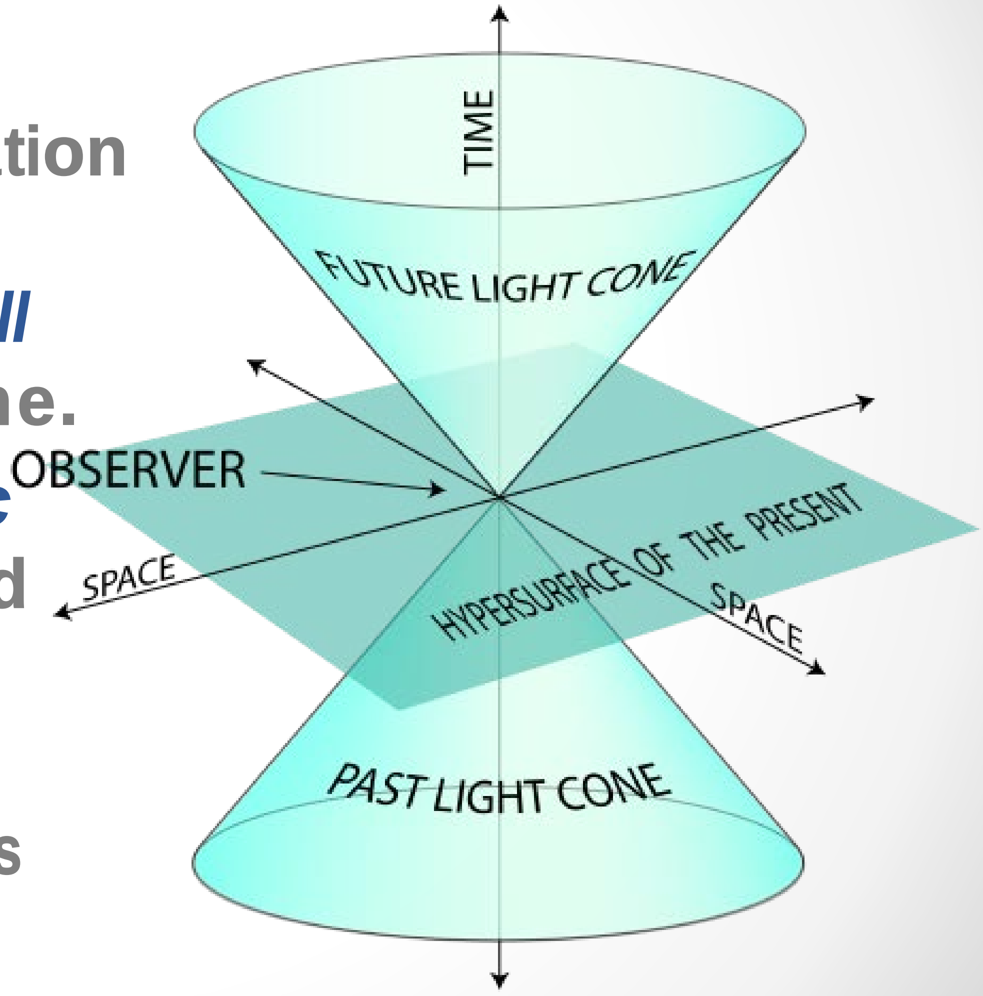
Gravity & Electromagnetism on the Null Cone

Maria Babiuc-Hamilton
Marshall University, WV

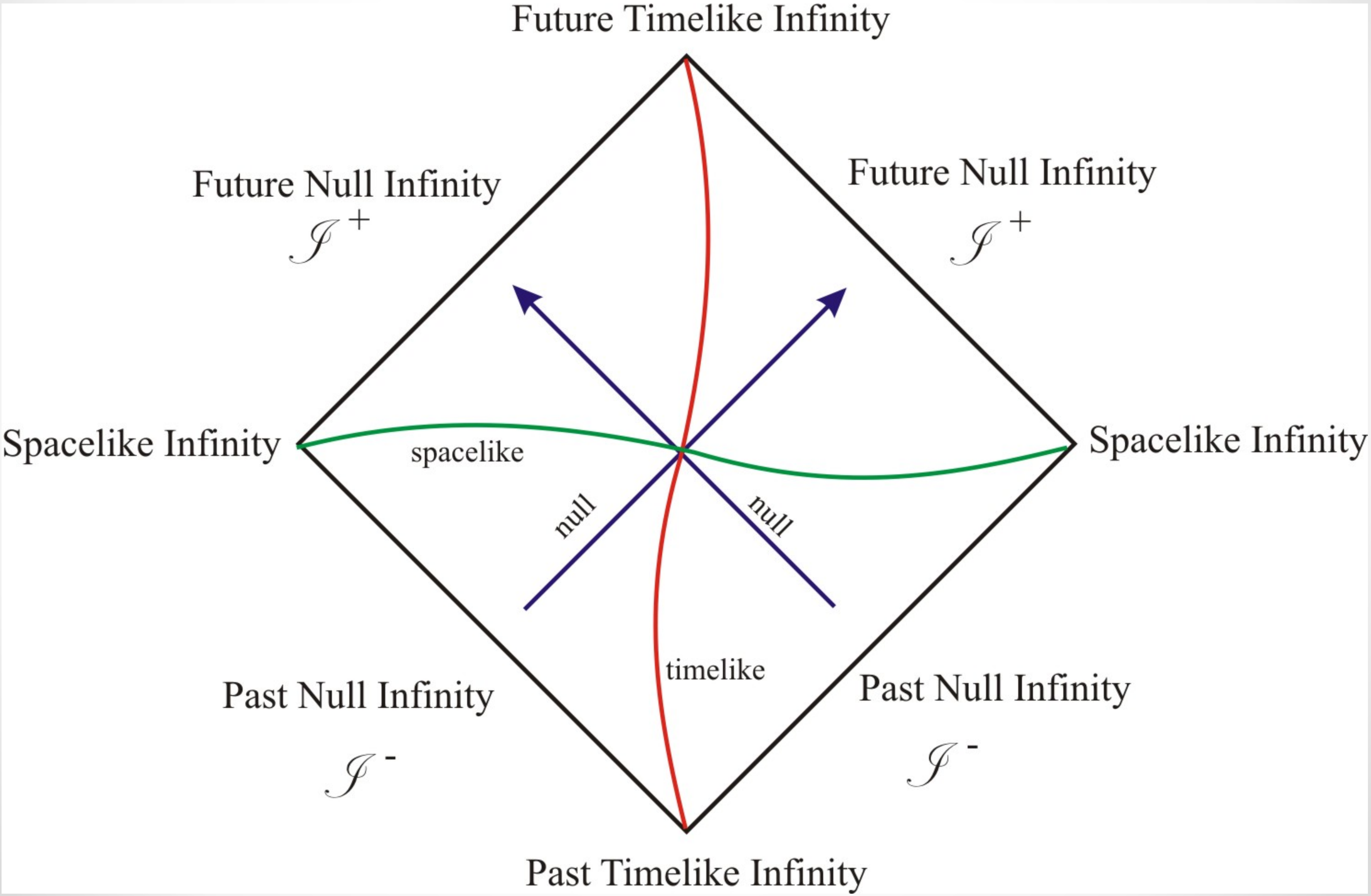
17th Eastern Gravity Meeting
May 16, 2014, WVU

The Null Cone

- Both gravitational and electromagnetic radiation travel along light rays, which are ***principal null directions*** in space-time.
- They are ***characteristic surfaces*** of Einstein and Maxwell equations.
- In characteristic coordinates, the field is described by ***ordinary differential equations***.



The Infinities

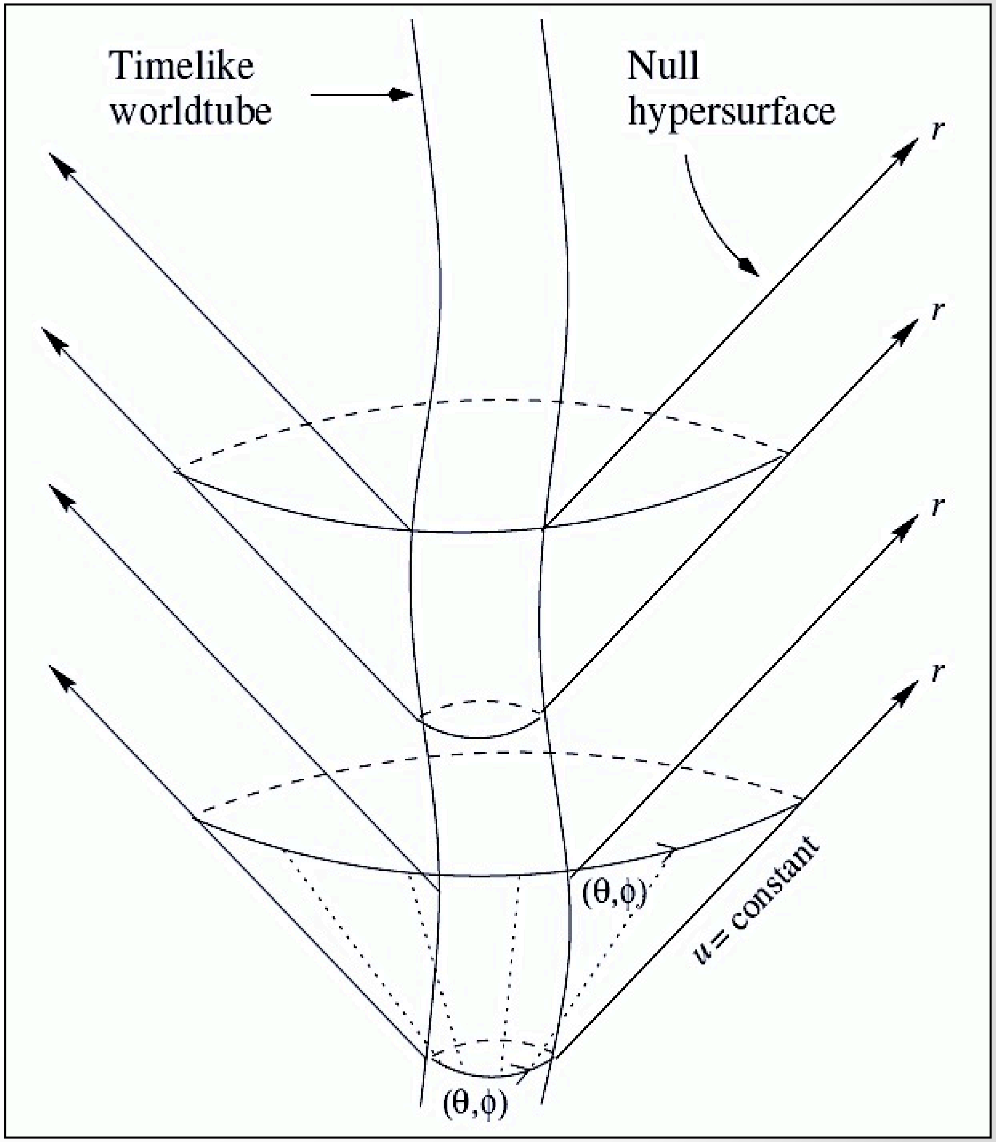


The Method

- Foliation of space-time by outgoing null rays ***u=t-r = const***
- Null grid **(*u, r, θ, φ*)**
- Infinity included by compactification

$$x = \frac{r}{R+r}, r \to \infty \Rightarrow x \to 1$$

- Combine the timelike close field with the characteristic far field.
- Evolve Einstein equations.
- Calculate the gravitational waveform at null infinity.
-



The Electrovacuum

- Cover the space-time with the null Sachs metric:

$$ds^2 = \left(-e^{2\beta}\frac{V}{r} + r^2U^2e^{2\psi} + r^2W^2e^{-2\psi}\right)du^2 - 2e^{2\beta}dudr$$

$$-2r^2Ue^{2\psi}dud\theta - 2r^2W\sin\theta e^{-2\psi}dud\varphi + r^2(e^{2\psi}d\theta^2 + e^{-2\psi}\sin^2\theta d\varphi^2)$$

- Choose a null gauge to describe the EM field:

$$F_{\alpha\beta} = 2D_{[\alpha}A_{\beta]};\ with\ A_\alpha = (A_0, 0, A_2, A_3)$$

- Write down all the field equations to be solved:

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta};\quad D_{[\delta}F_{\alpha\beta]} = 0;\quad D^\alpha F_{\alpha\beta} = 0$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2}g_{\alpha\beta}R;\quad T_{\alpha\beta} = \tfrac{1}{4\pi}\left(F^\rho_\alpha F_{\beta\rho} - \tfrac{1}{4}g_{\alpha\beta}F_{\rho\sigma}F^{\rho\sigma}\right)$$

$$C_{\alpha\beta\gamma\delta} = R_{\alpha\beta\gamma\delta} - \tfrac{1}{2}\left(g_{\alpha\gamma}R_{\beta\delta} + g_{\beta\delta}R_{\alpha\gamma} - g_{\beta\gamma}R_{\beta\delta} - g_{\alpha\delta}R_{\beta\gamma}\right) - \tfrac{1}{6}\left(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}\right)R$$

- $$D^\alpha G_{\alpha\beta} = 0;\quad D^\alpha T_{\alpha\beta} = 0;\quad D^\alpha C_{\alpha\beta\gamma\delta} = \tfrac{1}{2}\left(D_\gamma R_{\beta\delta} - D_\delta R_{\beta\gamma}\right)$$
-

The Simplifications

- ***Causality***: no inflow radiation from future.
- ***Vacuum***: the Ricci scalar curvature is zero.
- ***Axial symmetry*** will not restrict the generality.
- ***Linearity***: The parallax, and luminosity distance *r* along the rays agree.
- ***Euclidean topology***: flat space-time reasonably far from the source.

$$R = 0$$

$$e^{\beta} \cong 1$$

$$V \cong r - 2\mathrm{M}(u, \theta)$$

$$W \cong 0, U \cong \frac{1}{r^2} \Upsilon(u, \theta)$$

The Equations

The linearized Sachs metric with axial symmetry is:

$$ds^2 = \left(-1 + \tfrac{2}{r}M + \tfrac{1}{r^2}\Upsilon^2 e^{2\Psi}\right)du^2 - 2dudr - 2\Upsilon e^{2\Psi} dud\theta + r^2(e^{2\Psi}d\theta^2 + e^{-2\Psi}\sin^2\theta d\varphi^2)$$

The EM field is described by the Faraday tensor:

$$E_r = F_{01} = -\partial_r A_0;\ E_\theta = F_{02} = \partial_u A_2 - \partial_\theta A_0;\ E_\varphi = F_{03} = \partial_u A_3;$$

$$B_\varphi = F_{12} = \partial_r A_2;\ B_\theta = F_{13} = \partial_r A_3;\ B_r = F_{23} = \partial_\theta A_3$$

The main system is given by 9 equations:

$$J_0 = D^\alpha F_{\alpha 0} = 0; \quad J_2 = D^\alpha F_{\alpha 2} = 0; \quad J_3 = D^\alpha F_{\alpha 3} = 0$$

$$R_{11} - 8\pi T_{11} = 0;\ R_{12} - 8\pi T_{12} = 0;\ R_{13} - 8\pi T_{13} = 0;\ R_{22} - 8\pi T_{22} = 0;$$

$$R_{23} - 8\pi T_{23} = 0;\ R_{33} - 8\pi T_{33} = 0;$$

There are 5 supplementary conditions left:

$$R_{00} - 8\pi T_{00} = 0;$$

$$J_1 = D^\alpha F_{\alpha 1} = 0;\ \ R_{01} - 8\pi T_{01} = 0;\ R_{02} - 8\pi T_{01} = 0;\ R_{03} - 8\pi T_{03} = 0$$

The Questions

1. How to decouple the gravity from the EM field?
2. How to form the hierarchy between the evolution and hypersurface equations, for the integration?

- Ideally: with initial data for the gravitational field Ψ_0, and for the electromagnetic field (E_0, H_0), we calculate (Y_0, M_0) and integrate to (Ψ_1, E_1, H_1).

- Hierarchical integration:

1. hypersurface equations for the gravitational field:

$$R_{12} - 8\pi T_{12} = 0 \Rightarrow \Upsilon_0; \quad R_{23} - 8\pi T_{23} = 0 \Rightarrow M_0$$

1. Evolution equation for the EM components:

$$J_0 = D^\alpha F_{\alpha 0} = 0 \Rightarrow \partial^2_{ur} A_0; J_2 = D^\alpha F_{\alpha 2} = 0 \Rightarrow \partial^2_{ur} A_2; J_3 = D^\alpha F_{\alpha 3} = 0 \Rightarrow \partial^2_{ur} A_3$$

2. Evolution equations for the gravitational field Ψ

$$R_{33} - 8\pi T_{33} = 0 \Rightarrow \partial^2_{ur} \Psi$$

The Algorithm

- The integration of the evolution equations will be done with the new version of the marching algorithm integrator, that assures well-behavior.
- The The Weyl tensor will take account of the EM part:

$$C_{\alpha\beta\delta\sigma} = R_{\alpha\beta\delta\sigma} - \frac{1}{2}\left(g_{\alpha\delta}E_{\beta\sigma} + g_{\beta\sigma}E_{\alpha\delta} - g_{\alpha\sigma}E_{\beta\delta} - g_{\beta\delta}E_{\alpha\sigma}\right);$$

$$E_{\alpha\beta} = R_{\alpha\beta} - 8\pi T_{\alpha\beta}$$

Both the Weyl and Faraday fields are projected onto a quasi-normal tetrad to extract the scalars encoding the measurable values of the fields.

The Extensions

- Compactify the *r* coordinate.
- Carry the evolution to null infinity.
- Drop the axial symmetry.
- Express variables in series expansions of *r*.
- Implement the marching algorithms.
- Calculate the gravitational and electromagnetic waveforms.
- Include null neutrino dust.

The Benefits

- EM counterparts add to the knowledge of gravitational waves sources.
- Numerical relativity groups could develop their own homemade characteristic extraction modules.
- Other interesting phenomena
  - gravitational memory effect
  - formation of trapped surfaces
  - the problem of horizon