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Towards a Standardized Characteristic Extraction Tool

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Towards a Standardized Characteristic Extraction Tool

Maria Babiuc (Marshall University)
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Introduction

- Perturbative wave extraction at finite radius have errors:
  - gauge effects
  - nonradiative near fields
  - back reaction.
- Cauchy-characteristic extraction (CCE):
  - extends the simulation to future null infinity
  - where the waveform is computed in inertial coordinates.
- Usage of earlier implementations of CCE
  - waveforms from binary black hole simulations
  - waveforms from rotating stellar core collapse
  - exploration of the memory effect

Motivation

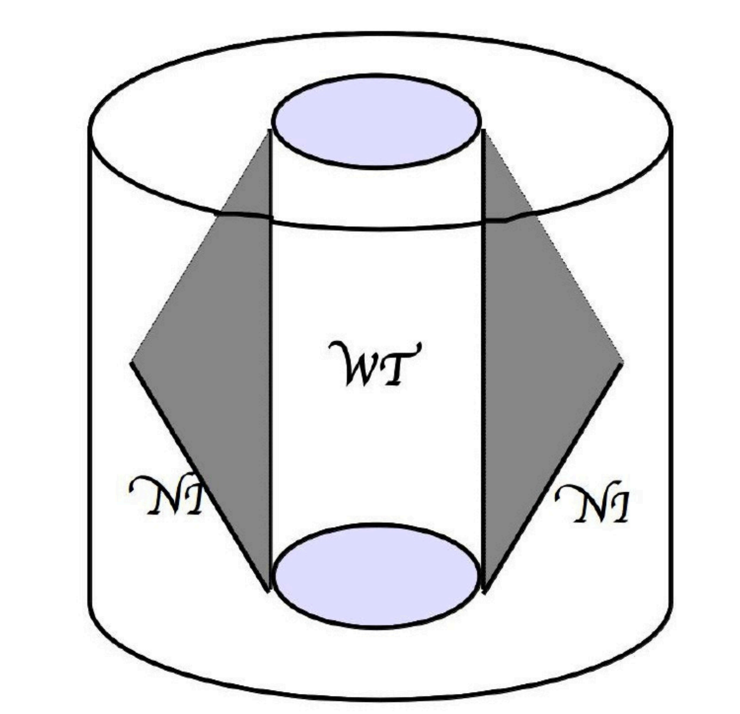
- Problems with the prior versions of the CCE:
  - Errors from the Cauchy evolution
    - introduced inconsistencies with the characteristic equations
    - affected the start-up algorithm
    - degraded the convergence rate.
  - Bugs introduced to remove second angular derivatives
    - in the radial start-up scheme of the auxiliary variables
    - a complex variable incorrectly declared to be a real variable.
  - Errors arising at the interpolation between the characteristic grid points and the extraction worldtube
    - A small stochastic component relative to the choice of grid, obscuring the results of convergence tests.

Contributions

- The new CCE module has been redesigned to be more accurate, efficient and easy to apply:
  - provides decoupling of the Cauchy and characteristic extraction errors
  - the start-up algorithm has been corrected to give clean second order accuracy with respect to grid size.
  - the auxiliary variables are initialized directly in terms of the main variables, avoiding Taylor expansions.
  - the interpolation errors have been reduced and tests have been applied to validate 2^{nd} order convergence.

Characteristic Initial Data

- Cauchy evolution supplies boundary data on the inner worldtube.
- The characteristic evolution extends the data to future null infinity, where the waveform is computed.
- The initial data for characteristic evolution should
  - suppress incoming radiation on the initial null hypersurface
  - provide continuity with the Cauchy data at the extraction worldtube while vanishing at plus null infinity.



$$J = \frac{J|_{x_E}(1-x)x_E}{(1-x_E)x}$$

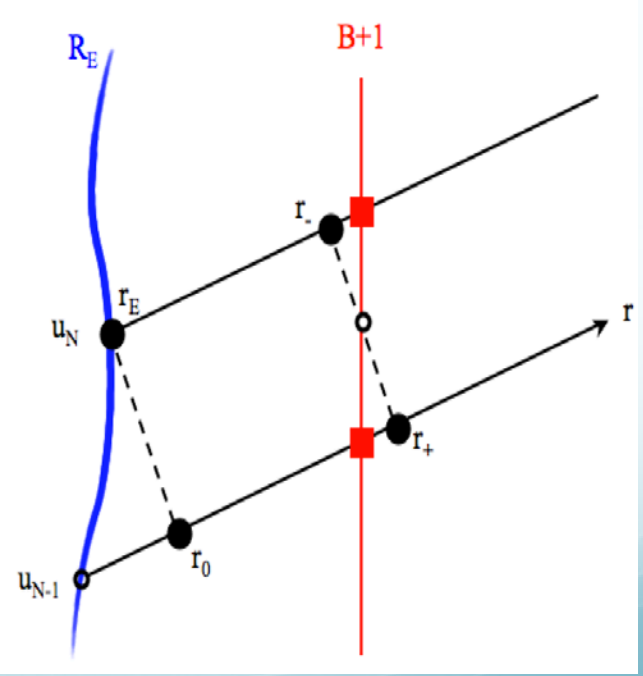
Start-up Algorithm

- The null parallelogram integration scheme is:

$$\Phi(u_n, r_-) = \Phi(u_n, r_E) + \Phi(u_{n-1}, r_+) - \Phi(u_{n-1}, r_0) + \frac{RHS(r_+ - r_0)\Delta u}{2}.$$

- The start-up value at the first active point is:

$$\Phi(u_n, r_-) = \frac{[\Phi(u_n, r_{B+1}) - \Phi(u_n, r_E)]r_-}{r_{B+1} - r_E}$$



- A stochastic grid-dependent source of second order error occurs in the start-up algorithm due to the location of the B +1 points.
- All the evolution variables measured at a finite radius exhibit second order convergence.
- Some asymptotic quantities have convergence rates intermediate between 1st and 2nd order.

Binary Black Hole Test

- Initial data: close quasicircular black-hole binary with orbital frequency MΩ= 0.050
- Cauchy evolution: LazEv code based on BSSN formulation and moving punctures, within Cactus framework and Carpet mesh refinement driver
- The perturbative waveform are extrapolated to infinity

$$\lim_{R\to\infty}[R\psi_4^{lm}(R,t)] = r\psi_4^{lm}(r,t) - \frac{(l-1)(l+2)}{2}\int_0^t dt\psi_4^{lm}(r,\tau)d\tau + \mathcal{O}(r^2)$$

- The characteristic extracted waveform is presented in terms of the Bondi news ***N*** or the Weyl tensor ***Ψ***

$$\Psi = \partial_u N \qquad \Psi_4 = -2\bar{\Psi} \qquad \Psi_4^0(u,x^A) = \lim_{r\to\infty} r\psi_4$$

Improved Convergence

- The (2;2) spherical harmonic mode at r = 80M for the metric variables near the peak of the signal at extraction radius R_E = 20M:

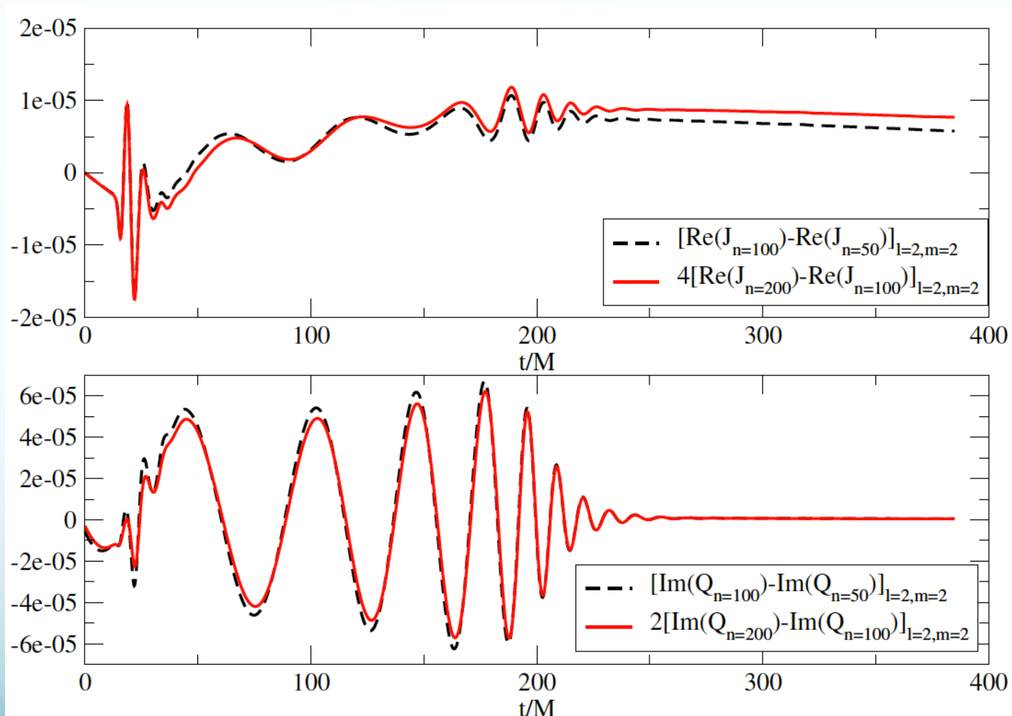
$Variable$	$Rate_{Re}$	$Rate_{Im}$
β	2.01	2.01
J	2.23	2.01
$J_{,x}$	2.03	2.33
Q	2.02	2.04
U	1.99	1.96
W	1.97	2.00

- The (2;2) spherical harmonic mode at null □ for the metric variables near the peak of the signal at extraction radius R_E = 20M:

$Variable$	$Rate_{Re}$	$Rate_{Im}$
β	2.01	2.01
J	1.80	2.18
$J_{,x}$	1.23	1.20
Q	1.63	1.03
U	1.99	1.96
W	1.95	1.11

Waveform Convergence

- Convergence plots for the (2;2) mode of the metric components at Scri (positive null infinity):



- Convergence rates of the (2;2) spherical harmonic mode for the Bond news ***N*** and Weyl component ***Ψ:***

$Variable$	$Rate_{Re}$	$Rate_{Im}$
N	1.59	1.56
$\partial_u N$	1.57	1.55
Ψ	1.16	1.14

- The convergence rates are affected by two factors:
  - the large number of terms involved in the calculation
  - dependence on radial one-sided derivatives at Scri.

Richardson Extrapolation

- The first order convergence results for the news ***N*** and Weyl component Ψ allows us to apply Richardson extrapolation.
- We use the results from the three different resolutions:

$$F_1 = f(\Delta),\ F_2 = F(2\Delta)\ F_4 = F(4\Delta)$$

- We construct second order accurate waveforms:

$$F_I = 2F_1 - F_2 \qquad F_{II} = 2F_2 - F_4$$

- We extrapolate to obtain 3[rd] order accurate waveforms:

$$F_E = \frac{8}{3}F_1 - 2F_2 + \frac{1}{3}F_4$$

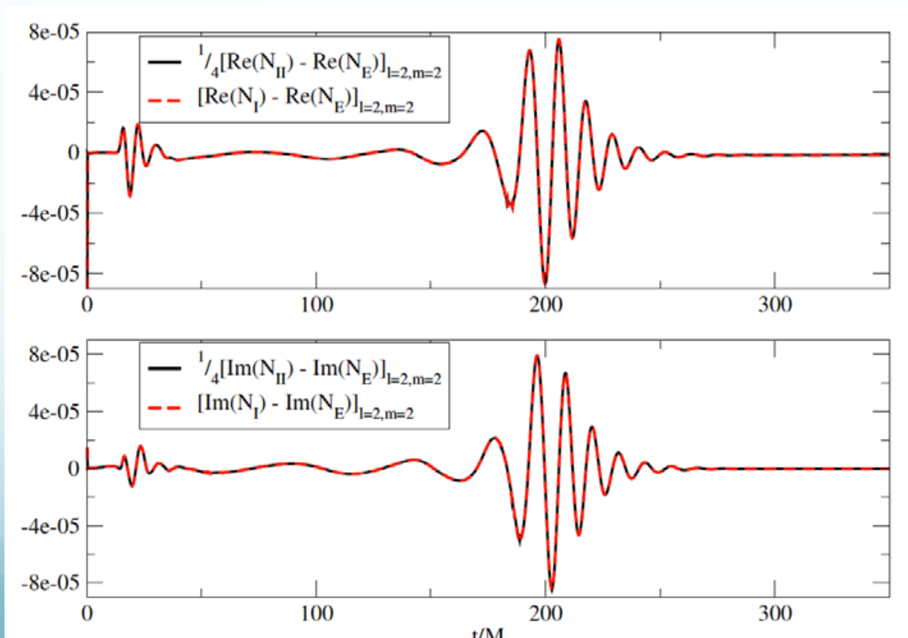
- We estimate the truncation errors in the News, Weyl component and perturbative waveform:

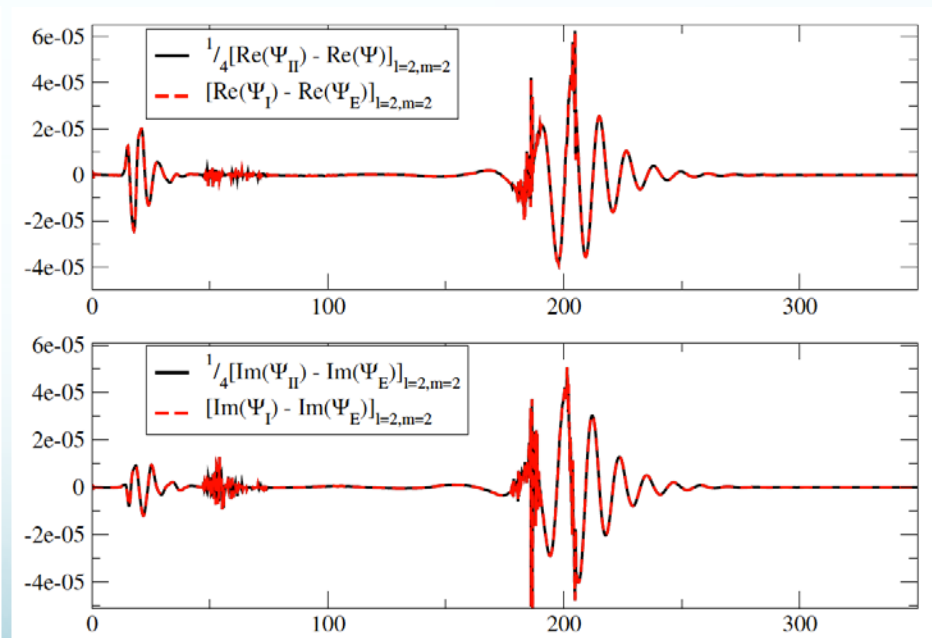
$$\delta N = N_I - N_E \qquad \delta\Psi = \Psi_I - \Psi_E \qquad \delta\psi_4 = (\frac{1}{2}r\psi_4 + \bar{\Psi})$$

Improved Waveform Convergence

- Plots of the rescaled errors of the real and imaginary parts of the (2;2) spherical harmonic mode for the news. The rescaled errors show that N_I and N_{II} are 2nd order accurate:

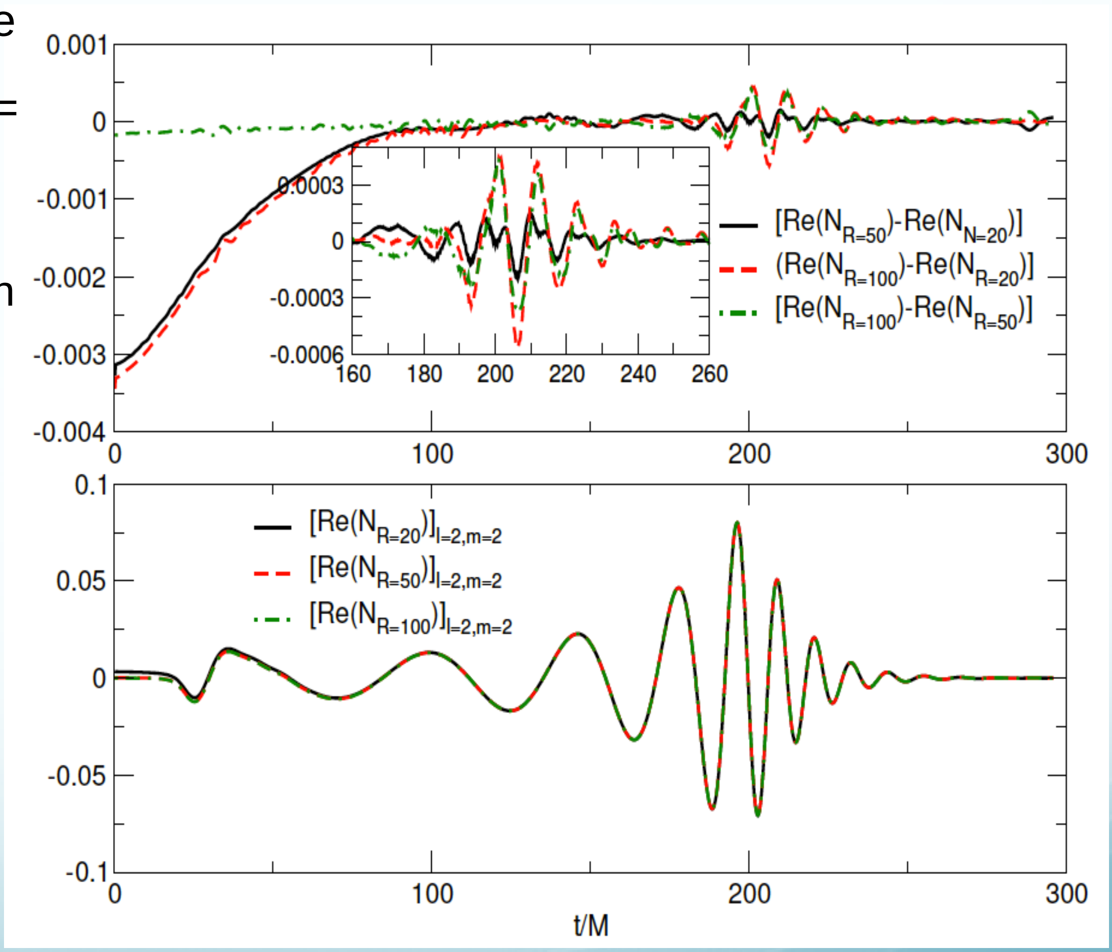
- The rescaled errors for the waveform Ψ(2;2) spherical harmonic mode. show that Ψ_I and Ψ_{II} are second order accurate. The second order error in Ψ contains more high frequency noise than for N:





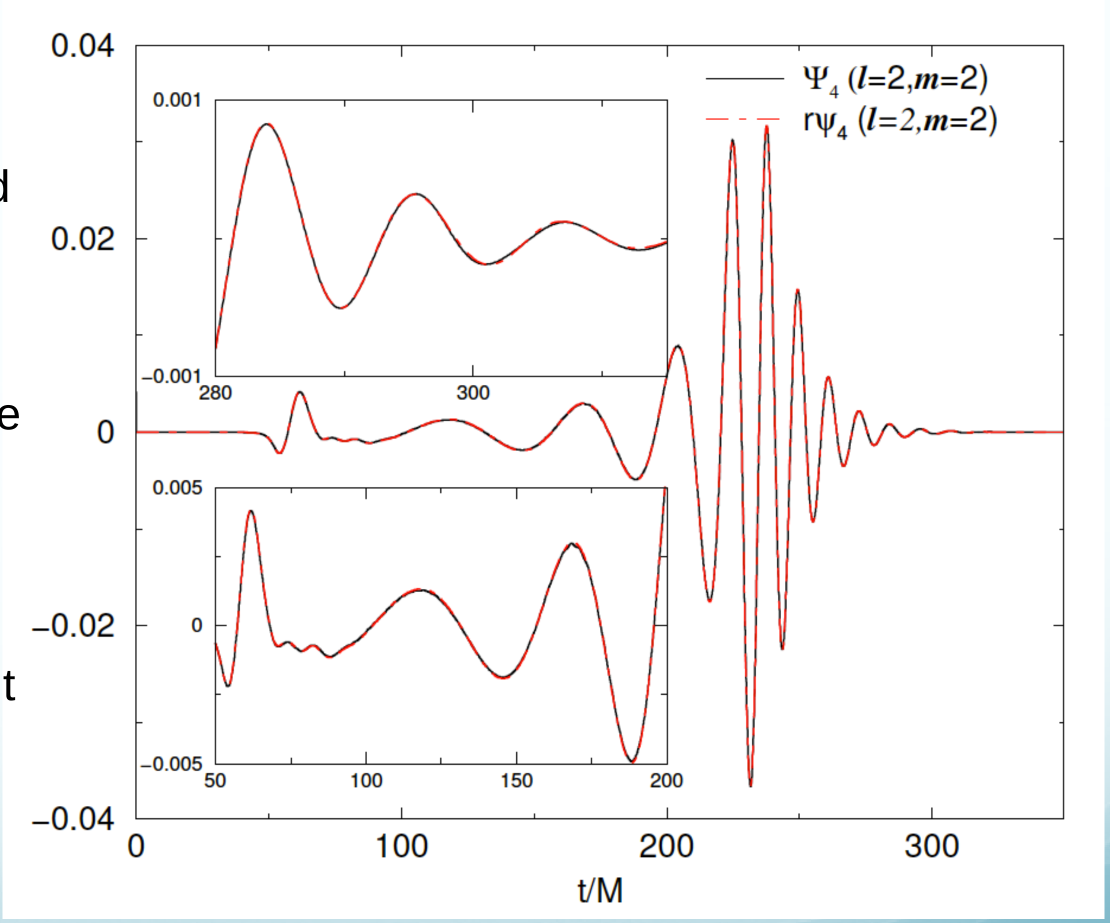
Waveform Extraction at Different Radii

- Plots of the dominant (2; 2) mode of Richardson extrapolated waveform for extraction radii R_E = 20M, 50M, and 100M:
  - The difference between R_E = 20M waveform and the other two due to a mismatch between the initial characteristic and Cauchy data, which decreases with large extraction radii.
  - The double hump results from non-trivial junk radiation in the initial Cauchy data.
  - The three waveforms are in good agreement in the inspiral and merger stage.
  - At the peak of the wave, the relative difference between waveforms is less than 0.6%.



Cauchy and Characteristic Waveform Comparison

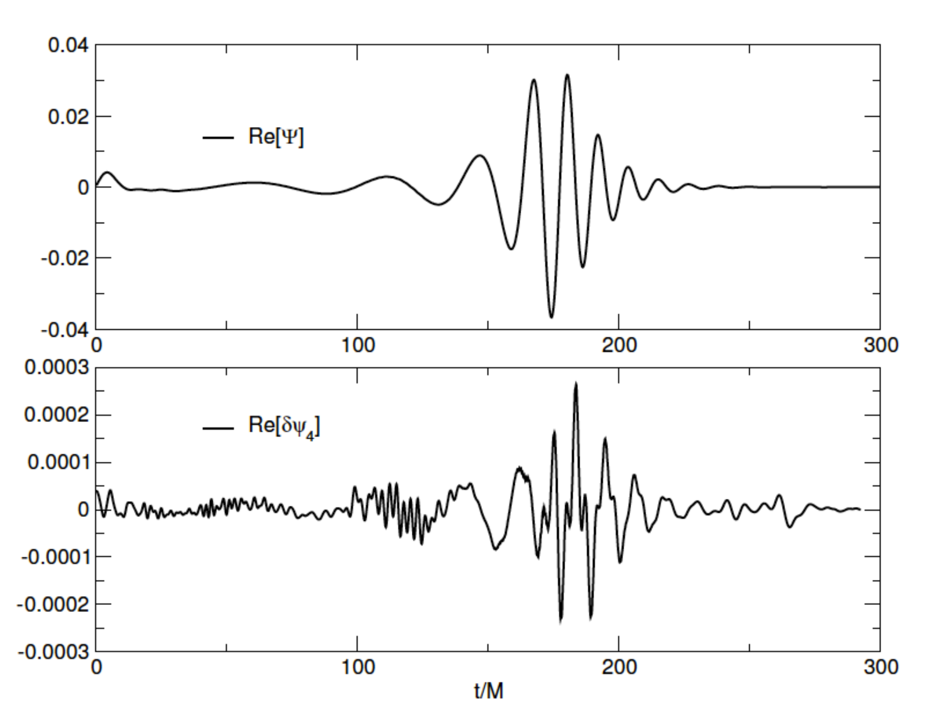
- Comparison of the (2; 2) dominant spherical harmonic mode for the characteristic and perturbative waveforms, extracted at R = 50M:
- Excellent agreement is shown, from the early stages, when the amplitude is small, and throughout the final ringdown.
- The perturbative extrapolation formula is essential to obtain this excellent phase agreement between the perturbative and characteristic waveforms.

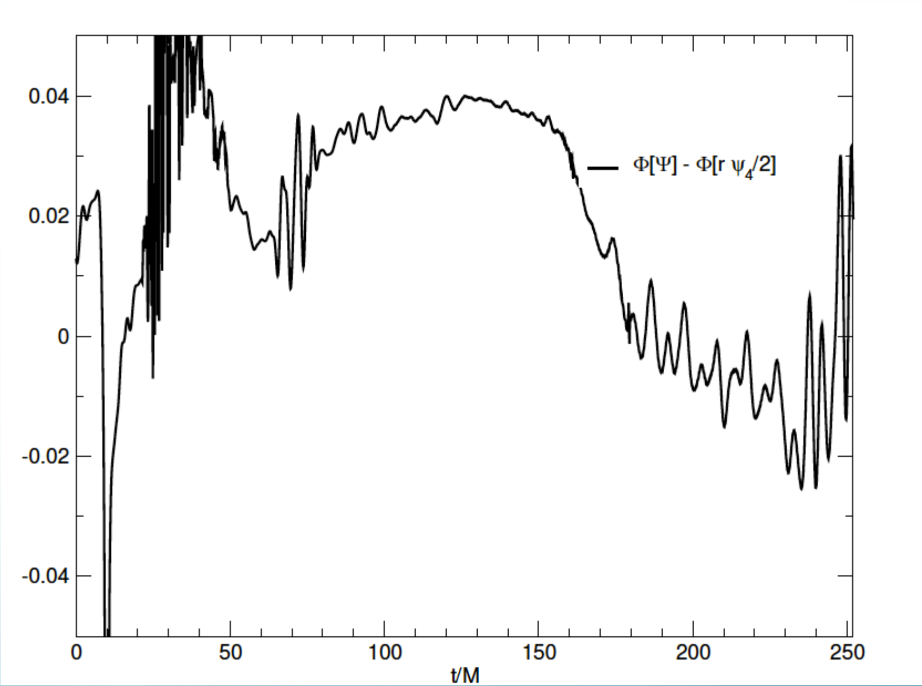


Cauchy and Characteristic Waveform Differences

- Plots of the characteristic waveform Ψ and the time dependence of the (2;2) spherical harmonic components of $\delta\psi_4$. The difference between perturbative and characteristic extraction is roughly 1% between the peak amplitudes of $\delta\psi_4$ and Ψ:

- Plot of the difference in phase Φ in radians, between the (2;2) components of the characteristic waveform Ψ and the perturbative waveform $\delta\psi_4/2$. Over the course of the simulation, their phases vary by less than 0.6 radians.





Advanced LIGO Accuracy Standards

- Accuracy standards required for application to advanced LIGO data analysis:
  - for detection

$$\mathcal{E}_k \le C_k\sqrt{2\epsilon_{max}}$$

  - for measurement

$$\mathcal{E}_k \le C_k\frac{\eta_c}{\rho} \quad \rho^2 = \int_0^\infty \frac{4|\hat{h}(f)|^2}{S_n(f)}df$$

- The time [illegible] numerical waveform with strain component *h(t)*:

$$\mathcal{E}_0 = \frac{||\delta h||}{||h||}$$

- The first time derivative corresponds to the error in ***N*** and the second time derivative to the error in ***Ψ*** component:

$$\mathcal{E}_1(ReN) = \frac{||\delta ReN||}{||ReN||}, \quad \mathcal{E}_1(ImN) = \frac{||\delta ImN||}{||ImN||}$$

$$\mathcal{E}_2(Re\Psi) = \frac{||\delta Re\Psi||}{||Re\Psi||}, \quad \mathcal{E}_2(Im\Psi) = \frac{||\delta Im\Psi||}{||Im\Psi||}$$

$$\mathcal{E}_2(Re\psi_4) = \frac{||Re\delta\psi_4||}{||Re\Psi||}, \quad \mathcal{E}_2(Im\psi_4) = \frac{||Im\delta\psi_4||}{||Im\Psi||}$$

Fulfillment of Advanced LIGO Requirements

- Error norms of the (2;2) spherical harmonic mode for the Bondi news ***N***, its counterpart N_Ψ (obtained by time integral of the Weyl component Ψ) and for the differences N_R obtained at extraction radii R_E = 20M, 50M and100M

$Variable$	Re	Im
$\mathcal{E}_1(N)$	8.761×10^{-4}	8.743×10^{-4}
$\mathcal{E}_1(N_\Psi)$	8.989×10^{-4}	8.947×10^{-4}
$\mathcal{E}_1(N_{\Delta R(20,100)})$	5.408×10^{-3}	5.549×10^{-3}
$\mathcal{E}_1(N_{\Delta R(50,100)})$	4.278×10^{-3}	4.513×10^{-3}

- The criterion for detection
  - ε_{max} = 0.005, 0.24 ≤ C_1 ≤ 0.8
  - E_1 ≤ 0.1C_1 ≤ 0.024
- This is satisfied *throughout the entire binary mass range* by CCE waveforms obtained from either the news or Weyl component.
  - In addition, the detection criterion is unaffected by choice of extraction radius.
- The criterion for measurement
  - η_{min} = 0.4, C_1 =0.24, ρ=100
  - E_1 =9.6 ×10^{-2}/ρ =9.6×10^{-4}
- This is also satisfied *throughout the entire binary mass range.*

Conclusions

- We introduced major improvements and corrections to prior versions the extraction tool.
- We tested the new CCE on the Cauchy evolution of the inspiral and merger of two equal mass non-spinning black holes.
- We presented convergence tests which demonstrate second order global accuracy of the evolution variables.
- We constructed third order accurate waveforms using the Richardson extrapolation, accuracy not possible with earlier versions.
- We assessed the accuracy of the waveforms with respect to the standards required for application to advanced LIGO data analysis.
- We proved that the numerical error introduced by CCE satisfies the time domain criteria for advanced LIGO detector.

Discussion

- Although the new CEE extraction tool contains major improvements and corrections of prior versions, there is still work to be done to improve the accuracy.
- The importance of accurate waveforms to the gravitational wave astronomy has created an urgency to make the characteristic waveform extraction a widely available tool.
- The module has been designed to provide a standardized waveform extraction tool for the numerical relativity community which will allow CCE to be readily applied to a generic Cauchy code.
- We are in the process of releasing the source code to the public as part of the Einstein Toolkit and we welcome applications to codes based different formulations of the Einstein equations and numerical techniques.

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$$\Psi_4^0(u, x^A) = \lim_{r\to\infty} r\psi_4$$

$$\hat{\Psi} = -(1/2)\bar{\psi}_4^0$$

$$\Psi = \partial_u N$$

$$\Psi = \frac{1}{2}\partial_u^2\partial_\ell J - \frac{1}{2}\partial_u J - \frac{1}{2}\eth L - \frac{1}{8}\eth^2(\eth\bar{L} + \bar{\eth}L) + \partial_u\eth^2 H$$

$$N = \frac{1}{2}\partial_u\partial_\ell J + \frac{1}{2}\eth^2(\omega + 2H)$$

$$N_\Psi = N|_{u=0} + \int_0^u \Psi du$$