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# Gravitational & Electromagnetic Waves on the Null Cone

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# * Gravitational & Electromagnetic Waves on the Null Cone

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# *Bondi Makes the News

* Bondi (1962) proved mathematically the existence of gravitational waves at null infinity.
* He found an exact solution of Einstein equations
* Within this metric, he calculated the ***loss of mass*** due to the emission of gravitational waves

$$ds^2 = -e^{2\beta}\frac{V}{r}du^2 - 2e^{2\beta}dudr$$
$$+ r^2 h_{AB}(dx^A - U^A du)(dx^B - U^B du)$$
$$N = N_+ + iN_X = \partial_t h_+ + \partial_t h_X$$

*The mass of a system is constant if and only if there is no **news**.*
*If there is **news**, the mass decreases as long as there are news.*

* We consider the coupled **Einstein-Maxwell system of** equations, on the Bondi null space-time metric.
* The choice of a null gauge field splits the coupled equations into spatial components, evolution equations, and conservation laws.

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}$$

$$T_{\alpha\beta} = \frac{1}{4\pi}\left(F_{\alpha\gamma} F^{\gamma}_{\beta} - \frac{1}{4} g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta}\right)$$

$$F_{\alpha\gamma} = 2D_{[\alpha} A_{\beta]};\ A^{u} = A_{r} = 0$$

$$D_{\beta} F^{\alpha\beta} - 8\pi J^{\alpha} = 0$$

# Gravitation and Light

# Main Equations for Light

- The Maxwell equation on the hyper-surface is $J^u$, with only radial and angular derivatives
- The evolution equations for the electromagnetic field are given by $J^B$
- The radial component is a conservation law, fulfilled everywhere if satisfied initially.

$$4\pi J^u = -\frac{D_C(h^{CD}\partial_r A_D)}{e^{2\beta}r^2} + \ldots$$

$$4\pi J^A = \frac{\partial_u(h^{AC}\partial_r A_C) + \partial_r(h^{AC}\partial_u A_C)}{e^{2\beta}r^2} + \ldots$$

$$4\pi J^r = -\frac{\partial_u(\partial_r A_u + U^D\partial_r A_D)}{e^{4\beta}}$$

$$- D_C\left(h^{CD}\partial_u A_D\right) + \ldots$$

# *The *Memory* Effect

* The last equation provides the information on the electric and magnetic parts of the null radiation ***memory*** effect: change in relative separation of two test particles.

$$D_B(h^{BC}E_C) + \frac{r^2}{e^{2\beta}} D_B(U^B E_r)$$

$$+ D_B \left[ h^{BC} \left( \frac{V}{r} B_C - U^C B_r \right) \right]$$

$$- \frac{r^2}{e^{2\beta}} D_B(U^B U^C B_C) =$$

$$4\pi e^{2\beta} r^2 J^r + \frac{r^2}{e^{2\beta}} + \ldots.$$

$$E_r = \partial_r A_u, E_C = 2\partial_{[C} A_{u]},$$

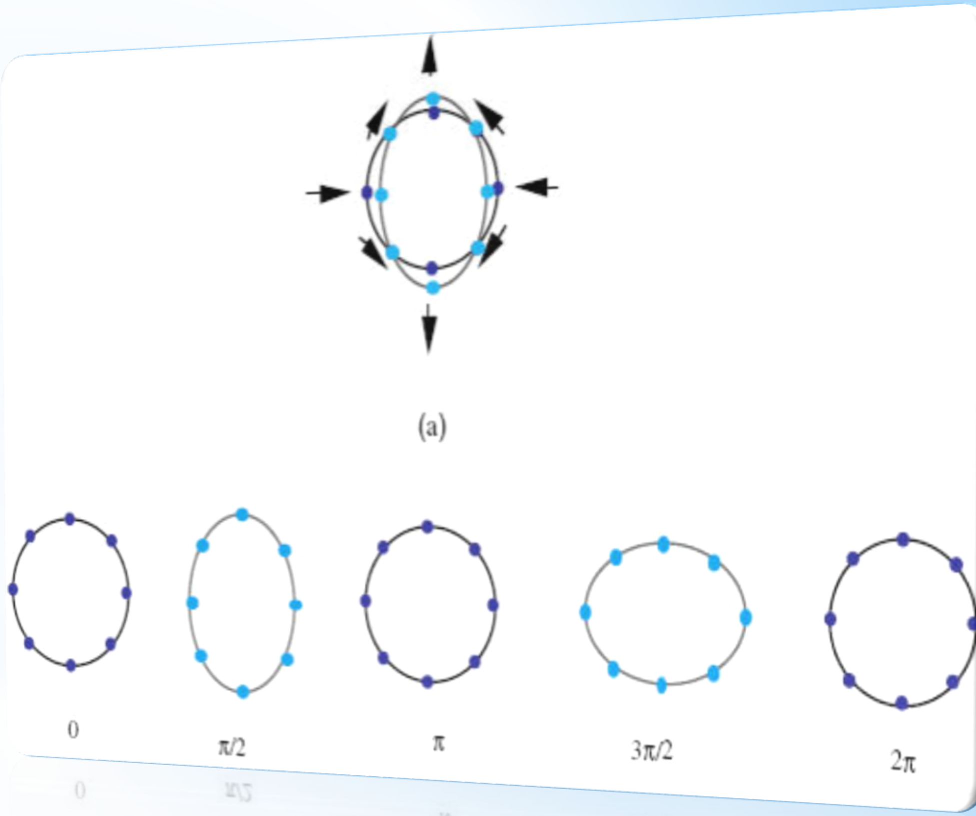
$$B_r = 2\partial_{[C} A_{B]}, B_C = \partial_r A_C$$

* Memory involves a process of physical changes in the structure of **receptors.**
  - Short-term or temporary
  - Long-term or permanent

# *What is Memory

# Gravitational Wave Memory

- The **memory** of a **gravitational-wave** burst is the **permanent** relative displacement of receptors.
  - Linear or **ordinary**, due to anisotropic source emission
  - Nonlinear or **null**, produced by the flux of gravitational or electromagnetic waves to propagating to null infinity



# * The *eth* Formalism

* Covers the unit 2D sphere with complex coordinates
* Tensors are decomposed in spin-weight scalars
* Express angular derivatives avoiding polar singularities
* We need the eth derivative of the 4-potential (null gauge)

$$h_{AB} = J\bar{q}_A\bar{q}_B/2 + \bar{J}q_Aq_B + Kq_{AB}/2$$

$$K = h_{AB}q^{AB}/2, \;\; J = h_{AB}q^Aq^B/2,$$

$$D_CA_B = D_{(C}A_{B)} + D_{[C}A_{B]}$$

$$D_CA_B = (\bar{q}_C\bar{q}_B\eth\mathcal{A} + q_Cq_B\bar{\eth}\bar{\mathcal{A}})/4$$

$$+ (q_C\bar{q}_B\bar{\eth}\mathcal{A} + \bar{q}_Cq_B\eth\bar{\mathcal{A}})/4$$

$$- (KA - J\bar{A})(q_{CB}\eth\bar{J} + \bar{q}_B\bar{q}_C\eth K)/4$$

$$- (K\bar{A} - \bar{J}A)(\bar{q}_C\bar{q}_B\eth J - q_Cq_B\eth\bar{J})/8$$

# Einstein-Maxwell Equations

- The Maxwell tensor splits into hyper-surface, evolution and conservation equations
- The complete algorithm proceeds in two steps:

1. Solve the hierarchical equations on the hypersurface
2. Integrate the metric and the 4-potential on the null cone.

$$\partial_r \beta = \mathcal{N}_\beta(h_{BC}) - 8\pi r T_{rr}$$

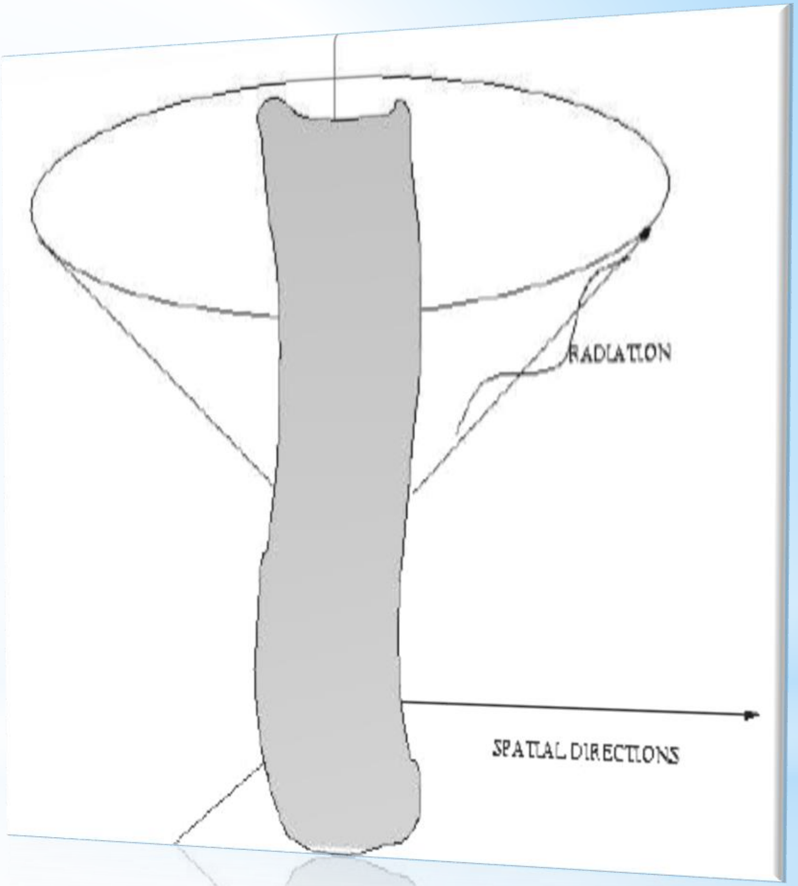
$$\partial_r(r^4 e^{-2\beta} h_{AB} \partial_r U^B) = \mathcal{N}_U(\beta, h_{BC}) - 8\pi r T_{ru}$$

$$\partial_r V = \mathcal{N}_V(U^A, \beta, h_{BC}) - 8\pi r T_{rB}$$

$$\partial_r\left(\partial_u(r h_{BC}) - \frac{V}{2r}\partial_r(r h_{BC})\right) = \mathcal{N}(U, \beta, h_{BC}) - 8\pi r T_{BC}$$

# *The Numerical Algorithm

* The null metric and 4-potential are reconstructed from a spectral decomposition and calculated on the world-tube by two coordinate transformations
* The Maxwell field is added to the Einstein equations and evolved on the null ray
* The gravitational waveforms are extracted at null infinity on inertial coordinates as ***News***



* The Kerr-Schild solution described a black hole with charge and spin, and is asymptotically flat
* For a non-spinning charged black hole we recover the Reissner-Nordstrom solution

$$g_{\mu\nu} = \eta_{\mu\nu} + F k_\mu k_\nu$$

$$ds^2_{KS} = -(1-F)du^2 - 2dudr - 2aF\sin^2\theta du d\theta$$
$$+ (r^2 + a^2\cos^2\theta)d\theta^2 + 2a\sin^2\theta dr d\phi$$
$$+ [a^2(1 + F\sin^2\theta) + r^2)sin^2\theta d\phi^2$$

$$F = \left(\frac{2Mr - Q^2}{r^2 + a^2\cos^2\theta}\right), \quad A_\mu = \frac{Q}{r^2 + a^2\cos^2\theta}k_\mu$$

$$k_\mu = \left(-1, 0, 0, a\sin^2\theta\right)$$

$$ds^2_{RN} = -(1-F)du^2 - 2dudr + r^2(d\theta^2 + \sin^2\theta d\phi)$$

$$F = \left(\frac{2M - Q^2 r}{r^2}\right), \quad A_\mu = \frac{Q}{r}(-1, 0, 0, 0)$$

# * Test Design

# *Adding to the News

* Next:

1. Develop and test the code
2. Run the code with a realistic configuration to extract GW and EM waveforms

* Study interesting phenomena:
  - Null GW and EM memory
  - Trapped surfaces, horizons