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A Generalization of the Difference of Slopes Test to Poisson Regression with Three-Way Interaction

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A GENERALIZATION OF THE DIFFERENCE OF SLOPES TEST TO POISSON REGRESSION WITH THREE-WAY INTERACTION

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APPROVAL OF THESIS/DISSERTATION

We, the faculty supervising the work of Melinda Bierhals, affirm that the thesis, Poisson Regression Model with Three-way Interaction. meets the high academic standards for original scholarship and creative work established by the Department of Mathematics and the College of Science. This work also conforms to the editorial standards of our discipline and the Graduate College of Marshall University. With our signatures. we approve the manuscript for publication.

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ABSTRACT

Linear regression models involving interaction can use the difference of slopes test to compare slopes for various situations. We will be generalizing this process to develop a procedure to compare rates in a Poisson regression model, allowing us to consider unbounded count data as opposed to continuous data. We will apply this process to an educational data set from a sample of students located in two different Los Angeles high schools. Our model will include a three-way interaction and address the following questions:

- Does language ability impact the relationship between math ability and attendance in the same way for males and females?
- Does gender impact the relationship between math ability and attendance in the same way for students with high language ability versus students with low language ability?

CHAPTER 1

MOTIVATION

Education is defined by the Oxford English Dictionary as "The systematic instruction, teaching, or training in various academic and nonacademic subjects given to or received by a child, typically at a school; the course of scholastic instruction a person receives in his or her lifetime." [8]. Education impacts our lives on a daily basis whether we are teaching ourselves or we are teaching a class.

My current interest in this field is in the improvement of student learning. The spark behind my educational research came from participating in a personality testing session. The activity was used to improve the dynamics within my school athletic team. Everyone on the team was to learn how to communicate better after learning about our personalities. After this activity, I noticed that there was a group of team members that all fell into the same personality category and they all felt the same way about math. Upon noticing this I decided to investigate this phenomenon further.

I decided to run a pilot study testing students' personalities to see if these had an influence on their math grades. Performing this research only increased my curiosity to see what other factors might shape a student's math grade. Multiple factors affect a student's education at the same time. For example, the student's self-esteem might have an effect on the relationship between their personality type and their math grade. After reading an article on interaction, written by Jeremy F. Dawson [2], I felt that I could expand his research on linear models to support interests in educational research.

Dawson used a linear model with three-way interaction to explore the effects of autonomy and experience on the relationship between training and job performance of employees in a manufacturing company. At first I thought I could apply this type of model to my pilot study. Upon further research I discovered that I would like to investigate a response variable that has a Poisson distribution as opposed to a normal distribution. Instead of using my own data, I will be applying my results to data from a sample of Los Angeles high school students.

CHAPTER 2

LINEAR REGRESSION

2.1 FUNCTIONAL AND STATISTICAL RELATIONS

A specific tool that Jeremy Dawson used to analyze possible factors that influence the relationship between job performance and training is called regression analysis. He started by using a linear regression model with two-way interaction and then moved to a linear regression model with three-way interaction. In order to understand the Poisson regression model, we must first take a look at linear regression.

We will begin by comparing a functional and statistical relation as described by Neter, Wasserman, and Kutner [7]. A *functional relation* can be represented by a simple equation for which X is the independent variable and Y is the dependent variable:

$$Y = f(X),$$

where for every input value of X there is a unique output value Y given by the function f. The graphical representation of a functional relation will show that every output value will fall onto the graph of functional relationship, creating a perfect relationship between the independent and dependent variable. A good example illustrated by the Department of Statistics at The Pennsylvania State University is the conversion relationship between temperature in degrees Celsius (C) and temperature in degrees Fahrenheit (F) [14]. This relationship is shown in Figure 2.1 and is represented by $F = \frac{9}{5}C + 32$.

Notice how this functional relation produces a linear relationship such that all of the observations of the relation fall onto the line of functional relationship $F = \frac{9}{5}C + 32$. While many relations can be modeled in this fashion there are other models that exist that do not follow this behavior. In a *statistical relation* not all of the observations will fall onto the line of relationship, but instead will be scattered around the line. Here the line of relationship has created what is called a line of best fit. This will allow the researcher to see a trend and enable them to make predictions on the basis of the data [1]. This can be illustrated by another example given by The

Pennsylvania State University and is shown in Figure 2.2 [14]:

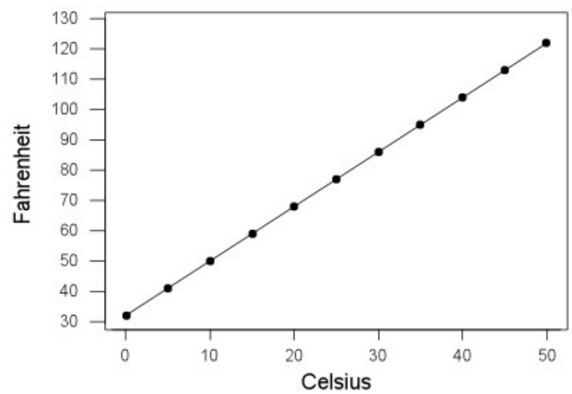

Figure 2.1: Conversion Relationship Between Celsius and Fahrenheit

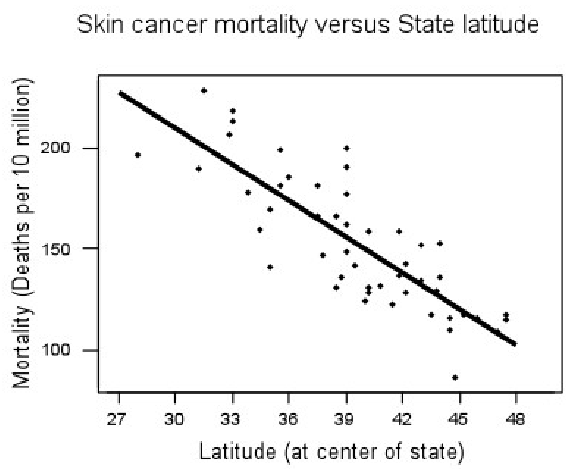

Figure 2.2: Relation Between Skin Cancer Mortality and State Latitude

The graph in Figure 2.2 represents the relationship between skin cancer mortality (in deaths per 10 million) and latitude (in degrees) at the center of each of the 50 U.S. states.

Now that we have determined the difference between functional and statistical relations we will be able to have a better understanding of the use of linear regression.

2.2 LINEAR REGRESSION TERMINOLOGY

Definition 1. *The Dependent Variable Y is also known as the Response Variable, and the Independent Variable X is also known as the Predictor Variable [7].*

Definition 2. *The diagram of a statistical relation is called a Scatter Diagram or Scatter Plot, where each point in the scatter diagram or scatter plot represents a trial or case [7].*

Definition 3. *The systematic relation of the means of the probability distributions Y to the level of X is called the Regression Function of Y on X.The Regression Curve is known as the graph of the regression function and will also be referred to as the Curve of Statistical Relationship [7].*

Definition 4. *The coefficients of the predictor variables will be known as the Parameters in the regression model [7].*

Definition 5. *The Scope of a model is a chosen interval that contains the values from the independent variables. The scope of a model is normally chosen by the design of the study or the range of the data involved in the study [7].*

2.3 SIMPLE LINEAR REGRESSION MODEL

Definition 6. *A simple linear regression model is one in which there exists only one independent variable, the parameters are not multiplied or divided by other parameters, and the independent variable is only to the first power. This model is also known as a first-order model [7].The simple linear regression model with one independent variable is as follows [7]:*

$$Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \tag{2.1}$$

where there exist two components, the systematic component $\beta_0 + \beta_1 X_i$ and the random component ϵ_i.

The systematic component and the random component are defined as follows: Y_i represents the value of the response variable in the ith trial, β_0 is a parameter that represents the Y-intercept of the regression line, β_1 is a parameter that represents the slope of the regression line, X_i represents the value of the predictor variable in the ith trial, and ϵ_i represents a random error term. Also it should be noted that the mean of ϵ_i is E$[\epsilon_i] = 0$ and the variance is σ^2. Here the regression function for 2.1 is:

$$E[Y] = \beta_0 + \beta_1 X. \tag{2.2}$$

The slope of the regression line β_0 represents the rate of change in the mean of the probability distribution of Y per unit increase of X [7]. When X = 0 is in the scope of the model, then the Y-intercept β_0 represents the mean of the probability distribution of Y at X = 0. If the scope does not include X = 0, then β_0 does not have any meaning in the model.

The basic idea behind a simple linear regression model is for it to be able to show that the dependent variable Y will vary in a systematic fashion with respect to the independent variable X, allowing the researcher to make predictions based on the given data. Graphically we can refer back to Figure 2.2 to see a scattering of points around the curve of statistical relationship [7].

2.4 ESTIMATION WITH METHOD OF LEAST SQUARES

Dawson used a linear regression model to analyze the relationships among the variables. The traditional approach to estimation in a linear regression model is the Method of Least Squares. To estimate a simple linear regression function it is necessary to find estimators of the parameters β_0 and β_1. Using the Method of Least Squares will require us to look at the deviation of Y_i from its expected value, for each sample observation (X_i,Y_i) [7]. Given by:

$$Y_i - (\beta_0 + \beta_1 X_i)$$

Graphically, when we look at our regression line we want to be able to see a line of best fit where the sum of the squares of the vertical distances from each point to the line is at a minimum as shown in Figure 2.3 [1].

More specifically, we will be finding values b_0 and b_1 that minimize the sum of n squared deviations, given by:

$$Q = \sum_{i=1}^{n} (Y_i - b_0 - b_1 X_i)^2$$

The values of b_0 and b_1 that minimize Q are the estimators for β_0 and β_1, respectively. These

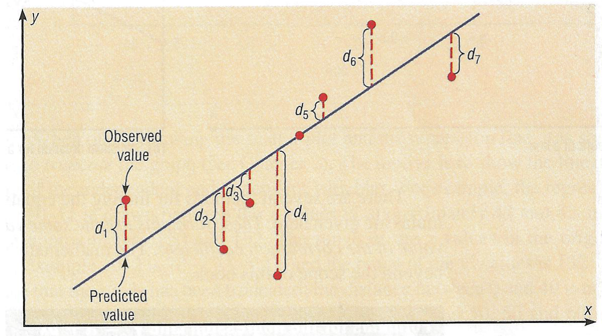

Figure 2.3: Line of Best Fit for a Set of Data Points

values b_0 and b_1 can be found by solving the following equations for b_0 and b_1 :

$$\Sigma Y_i = nb_0 + b_1 \Sigma X_i \tag{2.3}$$

$$\Sigma X_i Y_i = b_0 \Sigma X_i + b_1 \Sigma X_i^2 \tag{2.4}$$

When 2.3 and 2.4 are solved for b_0 and b_1 we obtain:

$$b_1 = \frac{\Sigma X_i Y_i - \frac{\Sigma X_i \Sigma Y_i}{n}}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}} = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2}$$

$$b_0 = \tfrac{1}{n}(\Sigma Y_i - b_1 \Sigma X_i) = \bar{Y} - b_1 \bar{X}$$

where $\bar{X}$ and $\bar{Y}$ represent the means of X and Y, respectively [7]. After finding b_0 and b_1 the regression function can then be estimated by the following:

$$\hat{Y} = b_0 + b_1 X$$

2.5 INTERACTION

We have just finished looking at a linear regression model with one predictor, X. While this model can be very helpful in making predictions of the response variable it may not be a suitable model for every situation. Another area of research includes looking at models with interaction, where the impact of one variable depends on the level of the other variable [9]. By looking at an example given by McGill University we can gain a better idea of the effects of interaction [11].

Suppose we consider a cholesterol lowering drug that is tested through a clinical trial. We are expecting a linear dose-response over a given scope of drug dose which will produce the simple linear model shown below.

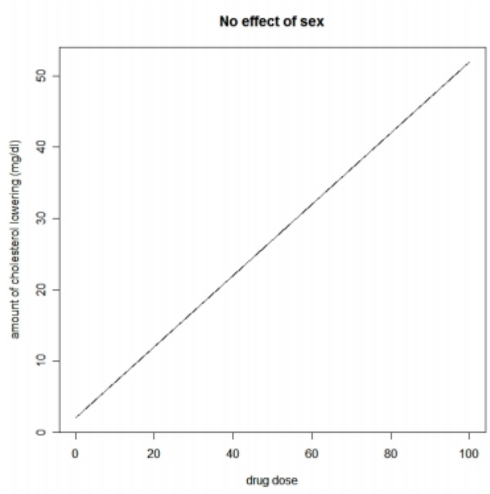

Figure 2.4: Dose-Response Over a Given Range of Drug Dose

Suppose that we expect men to respond at a higher level compared to women. There are a number of situations that can occur. Below are some graphs of two particular situations.

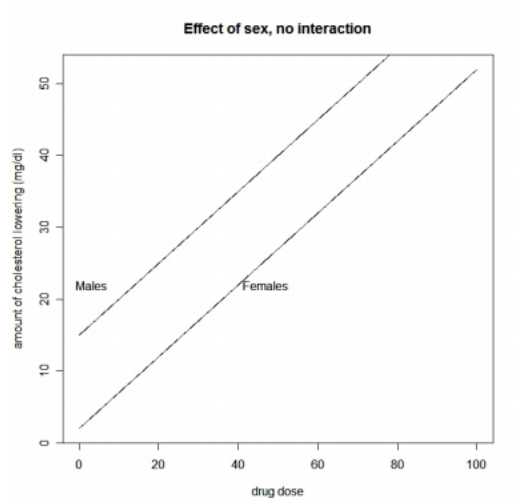

Figure 2.5: The Response Between Men and Women without Interaction.

Because the first situation shows no difference in the slopes of the lines for females and males, there is no presence of interaction here.

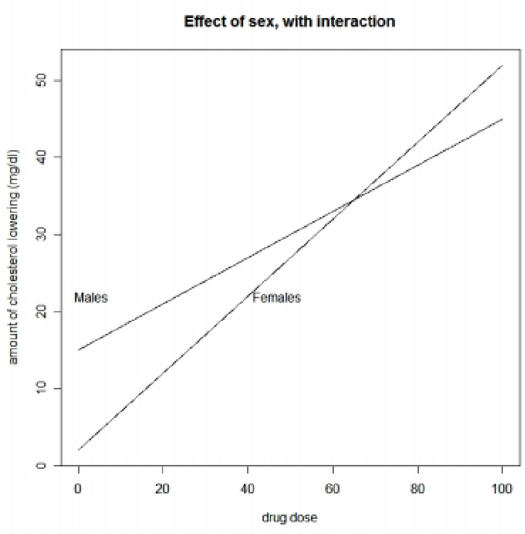

Figure 2.6: The Response Between Men and Women with Interaction.

Graphically, we are able to indicate the presence of interaction, but we will also need to be able to identify interaction in the actual models as well.

To be able to identify interaction in a model we will consider a first-order regression model that has two predictors X_1 and X_2 with no interaction [7]:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i \tag{2.5}$$

where, Y_i is the value of the response in the ith trial, X_{i1} and X_{i2} are the values of the predictors in the ith trial, β_0, β_1, and β_{i2} represent the parameters, and ϵ_i is the error term, with

$\epsilon_i = Y_i$ - $E[Y_i]$ This model is similar to a simple linear regression model in the fact that the relationship is linear. Also, 2.5 will have a regression function that represents a plane given by [7]:

$$E[Y] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \tag{2.6}$$

In this case β_0 represents the Y intercept of the regression plane. The other two parameters β_1 and β_2 will represent slopes. The parameter β_1 will represent the slope of the line relating Y to X_1 when X_2 is held constant. The parameter β_2 will represent the slope of the line relating Y to

X_2 whenX_1 is held constant. Since the effect of X_1 does not depend on the value of X_2 and the effect of X_2 does not depend on the value of X_1, there is said to be no interaction effect present in this model. Now let us consider a model that will have an interaction effect present. For example:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_{12} X_{i1} X_{i2} + \epsilon_i \tag{2.7}$$

This model contains two predictors X_1 and X_2. Notice that this model is similar to model 2.5. While both models contain two predictors, model 2.7 includes a cross-product of $\beta_{12} X_{i1} X_{i2}$ that represents the interaction term. In this model the interpretation of the slopes and the parameters β_1 and β_2 will not be as straightforward because of the presence of the interaction term. The following represent the slopes of the model:

- The slope of Y when X_2 is held constant is $\beta_1 + \beta_{12} X_2$ and the intercept is $\beta_0 + \beta_2 X_2$.
- The slope of Y when X_1 is held constant is $\beta_2 + \beta_{12} X_1$ and the intercept is $\beta_0 + \beta_1 X_1$.

Oftentimes the meanings of the coefficients are wrongly interpreted in the following ways. β_0=0 is taken as meaning that the intercept is zero, β_1=0 is taken as meaning that there is no linear relationship between X_1 and Y, and β_2=0 is taken as meaning that there is no relationship between X_2 and Y. In order for us to understand the concept of interaction we will need to consider a specific situation. Suppose that X_1 is the predictor we are interested in, and suppose that X_2 is the interacting variable. We would consider the follwoing two cases. If X_2=0 then $E[Y] = \beta_0 + \beta_1 X_1$, and if X_2=1, then $E[Y] = (\beta_0 + \beta_2) + (\beta_1 + \beta_{12}) X_1$. These models help us indicate the correct interpretation of the coefficients and their statistical significance. Here we see that β_0=0 means that the intercept is 0 *when* X_2*=0.* We see that β_1=0 means that there is no linear relationship between X_1 and Y *when* X_2*=0.* We also see that β_2=0 means that the value of X_2 has no effect on the intercept. Finally, we see that β_{12}=0 means that the value of X_2 has no effect on the slope, and therefore there is no presence of interaction. This last interpretation is the only one of the four that is generally used correctly.

If we would like to test the significance of the relationship between X and Y at specific predictor values, then we will need to consider the idea of centering [2]. For example, we will use model 2.7 and test the relationship of X_1 and Y at a particular value of X_2, denoted c. We will need to centerX_2 around the value of c by replacing X_2 with $X_2 - c$, which yields:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2(X_2 - c) + \beta_{12} X_1 (X_2 - c) \tag{2.8}$$

As a result of performing this centering the slope of Y is β_1 when X_2=c.

CHAPTER 3

GENERALIZED LINEAR MODELS

3.1 BASIC MODEL

Along with linear regression, Poisson regression refers to a specific type of generalized linear model. Generalized linear models are a broad class of models that include linear regression, ANOVA, Poisson regression, and log-linear models as well as many others [12]. Here we will discuss the makeup of any generalized linear model as described in an online article from Pennsylvania State University [12] and *Generalized Liner Models* by McCullagh and Nelder [6]. These models have three components that include a random component, a systematic component, and a link function:

$$f(\theta) = \beta_0 + \beta_1 X_1 \tag{3.1}$$

where,

- Random Component: contains the probability distribution of the response variable (Y), where the components are independently distributed.
- Systematic Component: specifies the predictor variables $(X_1, X_2, ..., X_k)$ that create a predictor vector. The predictors will have parameters $\beta_1, ..., \beta_p$ that are unknown and will need to be estimated.
- Link Function: explains how the expected value of the response relates to the linear predictor. This is the link between the random and systematic components.

In the case of the Poisson regression, we will need to use maximum likelihood estimation to estimate our parameters, instead of the method of least squares.

3.2 MAXIMUM LIKELIHOOD ESTIMATION

We have discussed the method of least squares used to estimate parameters in a linear model, but now it is time to discuss the approach used to estimate the parameters in a Poisson regression. Since the inferences associated with ordinary least squares reguire the the response variable to have a normal distribution and this restriction is not present in our generalized linear model, the use of ordnary least squares estimation would not be approprate. Instead, we use the method of maximum likelihood estimation. This method will provide us with a good estimate of the unknown paramter Θ that maximizes the probabilty, or the likelihood, of getting the data we observed [13]. The likelihood of a sample with continuous random variables $X_1, X_2 \ldots, X_n$ is defined to be the joint density function evaluated at $x_1, x_2, , x_n$,where their distribution depends on the parameter Θ. The likelihood function will be defined as follows:

$$L(\Theta) = L(\Theta|x_1, x_2, \ldots, x_n) = f(x_1, x_2, \ldots, x_n|\Theta) = f(x_1|\Theta) \times f(x_2|\Theta) \cdots \times f(x_n|\Theta)$$

The method of maximum likelihood will find the value of Θ that maximizes L(Θ). To find the value of Θ that maximizes L(Θ) we will do the following:

- find the log likelihood ln L(θ),

  note: Calculating the derivative of products can be a daunting task. We can simplify this process by taking the derivative of the natural logarithm of Θ, which is called the log likelihood function. Calculating the derivative of the log likelihood function will result in the same maximum likelihood estimate, because the natural log is an increasing function.

- calculate the derivative of L(Θ) with respect to Θ,
- set that derivative equal to zero, and
- solve the equation for Θ.

We will then call this value of Θ the maximum likelihood estimator, $\hat{\Theta}$.

Understanding the process used by the Method of Maximum Likelihood Estimation is important because it will be used to find the parameter estimates of our Poisson regression model with three-way interaction.

CHAPTER 4

SIMPLE POISSON REGRESSION MODEL

Now we will consider the nonlinear Poisson regression model. But, before we talk about the model we will define the Poisson distribution.

4.1 POISSON DISTRIBUTION

The Poisson distribution is often used to provide a good model for the probability distribution of the number Y of rare events that occur in time, space, or volume where λ is the average value of Y [15].

Definition 7. *A random variable Y is said to have a Poisson probability distribution if and only if*

$$f(y) = \frac{\lambda^y}{y!}e^{-\lambda} \tag{4.1}$$

where y= 0,1, 2,..., and $\lambda > 0$. It should also be noted that the mean and variance are $E[y] = \lambda$ and $\sigma^2[y] = \lambda$.

4.2 REGRESSION MODEL

As stated before, the Poisson regression model is a special case of a generalized linear model. Therefore, it is comprised of three components, the random, systematic, and the link.

- Random Component: response variable Y is a count and has a Poisson distribution.
- Systematic Component: specifies the predictor variables $(X_1, X_2, ..., X_k)$ that will have parameters $\beta_1, ..., \beta_k$ that are unknown and will need to be estimated.
- Link Function: natural log.

$$\ln \lambda = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k \tag{4.2}$$

Oftentimes the Poisson regression model is used to interpret the predictor coefficients in the form of rates [4] . These rates are used to describe the effect of a one unit increase in the predictor. To be able to interpret the coefficients as rates we must take the difference between the log of expected counts [4].

Below is an illustration for the interpretation of rates in the simple Poisson regression model $\ln \lambda(x) = \beta_0 + \beta_1 X$:

$$\ln \lambda_{x+1} = \beta_0 + \beta_1(X+1) \qquad\qquad \ln \lambda_x = \beta_0 + \beta_1 X$$

$$\begin{gathered}
\ln \lambda_{x+1} - ln\lambda_x = ln\left(\frac{\lambda_{x+1}}{\lambda_x}\right) \\
[\beta_0 + \beta_1(X+1)] - [\beta_0 + \beta_1 X] = ln\left(\frac{\lambda_{x+1}}{\lambda_x}\right) \\
\beta_1 = ln\left(\frac{\lambda_{x+1}}{\lambda_x}\right) \\
e^{\beta_1} = \left(\frac{\lambda_{x+1}}{\lambda_x}\right)
\end{gathered} \tag{4.3}$$

Thus e^{β_1} represents the percent increase in the expected count per unit of increase in the predictor [4]. When $x = 0$, then $\lambda(0) = e^{\beta_0}$; thus e^{β_0} is the intercept of the model. This concept will be illustrated further in our example with the three-way interaction Poisson model.

CHAPTER 5

POISSON REGRESSION MODEL WITH THREE-WAY INTERACTION

We will now apply the previous disscussions we had about interaction and the process of centering from the simple linear regression model along with the process of interpreting rates from the simple Poisson model. Recall from our discussion about interaction that we referred to a two-way interaction linear model 2.7 that contained two independent variables X_1 and X_2. In this case we will be considering a nonlinear regression model with one more level of interaction included, but the idea of interaction will be the same.

Definition 8. *A Poisson regression model with three-way interaction is defined as:*

$$\ln \lambda = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 + \beta_{123} X_1 X_2 X_3 \tag{5.1}$$

where $X_1, X_2,$ and X_3 are the predictors and $\beta_1, \beta_2, \beta_3, \beta_{12}, \beta_{13}, \beta_{23},$ and β_{123} are the parameters.

At this point we will manipulate the model slightly to make it easier to determine one of the rates. This same strategy was used on model 2.7 to find the slopes with respect to X_1 and X_2. First we will choose to hold X_2 and X_3 constant and call X_1 our target predictor. By doing this we can group together the terms including X_1:

$$\begin{aligned}
\ln \lambda &= (\beta_1 X_1 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{123} X_1 X_2 X_3) + (\beta_0 + \beta_2 X_2 + \beta_3 X_3 + \beta_{23} X_2 X_3) \\
&= X_1(\beta_1 + \beta_{12} X_2 + \beta_{13} X_3 + \beta_{123} X_2 X_3) + (\beta_0 + \beta_2 X_2 + \beta_3 X_3 + \beta_{23} X_2 X_3)
\end{aligned}$$

Here the X_1 coefficient is $(\beta_1 + \beta_{12} X_2 + \beta_{13} X_3 + \beta_{123} X_2 X_3)$ and the value of $(\beta_0 + \beta_2 X_2 + \beta_3 X_3 + \beta_{23} X_2 X_3)$ is considered the constant.

One of our goals will be to compare the rates when we fix the value of X_2 and vary the value of X_3. The other goal is to compare rates when we fix the value of X_3 and vary the value of X_2. We will proceed by showing the case of fixing X_2.

Specifically, we will fix X_2 at X_2= a and vary the value of X_3 from X_3= b to X_3= c. First we will replace X_2 with $X_2 - a$:

$$\ln\lambda = X_1\left\{\beta_1 + \beta_{12}(X_2 - a) + \beta_{13}X_3 + \beta_{123}(X_2 - a)X_3\right\} + \left\{\beta_0 + \beta_2(X_2 - a) + \beta_3 X_3 + \beta_{23}(X_2 - a)X_3\right\}$$

Then we will evaluate the model at X_2=a and X_3=b:

$$\begin{aligned}\ln\lambda &= X_1\left\{\beta_1 + \beta_{12}(a - a) + \beta_{13}(b) + \beta_{123}(a - a)(b)\right\} + \left\{\beta_0 + \beta_2(a - a) + \beta_3(b) + \beta_{23}(a - a)(b)\right\} \\ &= X_1\left\{\beta_1 + \beta_{13}(b)\right\} + \left\{\beta_0 + \beta_3(b)\right\}\end{aligned}$$

Next we will evaluate the model at X_2=a and X_3=c:

$$\begin{aligned}\ln\lambda &= X_1\left\{\beta_1 + \beta_{12}(a - a) + \beta_{13}(c) + \beta_{123}(a - a)(c)\right\} + \left\{\beta_0 + \beta_2(a - a) + \beta_3(c) + \beta_{23}(a - a)(c)\right\} \\ &= X_1\left\{\beta_1 + \beta_{13}(c)\right\} + \left\{\beta_0 + \beta_3(c)\right\}\end{aligned}$$

Since we want to test the equality of the rates our null hypothesis would be:

$$H_0 : e^{\beta_1 + \beta_{13}(b)} = e^{\beta_1 + \beta_{13}(c)},$$

which is equivalent to:

$$H_0 : \beta_1 + \beta_{13}(b) = \beta_1 + \beta_{13}(c)$$

which is in turn equivalent to:

$$H_0 : \beta_{13} = 0$$

Similarly, if we fix X_3 and let X_2 vary, we will test $H_0 : \beta_{12}$=0.

CHAPTER 6

Example

The relationship that we will be investigating is the relationship between math ability and attendance. Specifically, we will address the following two questions:

- Does language ability impact the relationship between math ability and attendance in the same way for males and females?
- Does gender impact the relationship between math ability and attendance in the same way for students with high and low language ability?

The data to be used here were selected from two senior high schools in the Los Angeles area. The student records we will be using contain 100% of the students at each of the schools who met all of the following criteria, as described by Phil Ender [3]:

- They were in the 9th grade in the 1995 Fall semester.
- They received a mark (grade) in first semester Algebra in the 1995 Fall semester and a mark (grade) in a mathematics course in the preceding Spring semester.
- They received a mark (grade) in an English course in both the 1995 Fall semester and the preceding Spring semester.
- They had a California Test of Basic Skills (CTBS) combined Mathematics and combined Language score for the 1995 calendar year. Both scores are recorded in percentile ranks (PR) and normal curve equivalence scores (NCE). The mathematics normal curve equivalence scores will be denoted as mathnce, and the language normal curve equivalence scores will be denoted as langnce.

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To help assess the fit of the model, we can use the chi-squared goodness-of-fit test. This assumes the deviations follows a chi-square distribution with degrees of freedom equal to the model residual [5]. From our Goodness of Fit output, we can see with 308 degrees of freedom and a value of 2656.3710 that a p-value can be calculated as approximately zero as shown in 6.1.

Criteria For Assessing Goodness Of Fit			
Criterion	DF	Value	Value/DF
Deviance	308	2196.5562	7.1317
Scaled Deviance	308	2196.5562	7.1317
Pearson Chi-Square	308	2656.3710	8.6246
Scaled Pearson X2	308	2656.3710	8.6246
Log Likelihood		1501.2620	
Full Log Likelihood		-1528.9759	
AIC (smaller is better)		3073.9519	
AICC (smaller is better)		3074.4209	
BIC (smaller is better)		3103.9978	

Table 6.1: Criteria For Assessing Goodness of Fit

With this p-value we can conclude that the model does not fit well at all because the goodness-of-fit chi-squared test is statistically significant. This is very likely due to the obvious overdispersion, and a model with another distribution such as the negative binomial might solve this problem. This provides an interesting possibility for future research.

In order to achieve a joint significance level of at most 5%, we apply the Bonferroni Adjustment and use $\alpha = \frac{0.05}{4} = 0.0125$ for each individual comparison [10].

Recall that our Poisson regression model with three-way interaction is as follows:

$$\ln\lambda = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_{12} X_1 X_2 + \beta_{13} X_1 X_3 + \beta_{23} X_2 X_3 + \beta_{123} X_1 X_2 X_3 \tag{6.1}$$

Parameter	Label
X_1	mathnce
X_2	langnce
X_3	sex
X_{12}	math·lang
X_{13}	math·sex
X_{23}	lang·sex
X_{123}	math·lang·sex

Table 6.2: Parameter Labels

Our model will be used to process the data with a response variable of attendance and a target predictor of mathnce. a reference for representation of each one of the parameters that will be used.

To test the impact of language ability we will consider high and low language scores of 68.0030042 and 32.1245876, which are one standard deviation above and below the mean language score.

The MEANS Procedure		Analysis Variable : langnce		
N	Mean	Std Dev	Minimum	Maximum
316	50.0637959	17.9392083	1.0071000	98.9929000

Table 6.3: Analysis of Language Normal Curve Equivalence Scores

After applying formula 4.3 to our model we found the following rates:

	Male	Female
Language High	ln λ = -0.0084X1 + 1.8760	ln λ = 0.0090X1 + 1.3209
Language Low	ln λ = -0.0155X1 + 2.3352	ln λ = 0.0054X1 + 1.9990

$e^{-0.0084} = 0.9916$	Expected number of absences decreases by about 1% for each 1-unit increase in mathnce.
$e^{0.0090} = 1.0090$	Expected number of absences increases by almost 1% for each 1-unit increase in mathnce.
$e^{-0.0155} = 0.9845$	Expected number of absences decreases by about 1.5% for each 1-unit increase in mathnce.
$e^{0.0054} = 1.0054$	Expected number of absences increases by about 1% for each 1-unit increase in mathnce.

Table 6.4: Comparison of Rates

Below is a table with the appropiate p-values for each comparison.

Predictor Variables	Variable Levels	Centering of Model	Parameter	p-value
X_2	Language Low	$X_2 \rightarrow X_2$ -32.1245876	β_{13}	< 0.0001
	Language High	$X_2 \rightarrow X_2$-68.0030042	β_{13}	0.0006
X_3	Male	$X_3 \rightarrow X_3$ -0	β_{12}	0.0714
	Female	$X_3 \rightarrow X_3$-1	β_{12}	0.4370

Table 6.5: P-Value Results

Comparison 1: Low Language Scores comparing Males and Females

Since the p-value for β_{13} is <0.0001, which is smaller than $\frac{\alpha}{4} = 0.0125$, we can conclude that there is a difference in the rate at which math ability affects attendance for males with low language skills versus females with low language skills.

Comparison 2: High Language Scores comparing Males and Females

Since the p-value for β_{13} is 0.0006, which is smaller than $\frac{\alpha}{4} = 0.0125$, we can conclude that there is a difference in the rate at which math ability affects attendance for males with high language skills versus females with high language skills.

Comparison 3: Males comparing Low and High Language Skills

Since the p-value for β_{12} is 0.0714, which is larger than $\frac{\alpha}{4} = 0.0125$, we can conclude that there is no evidence of a difference in the rate at which math ability affects attendance for males with low language skills versus males with high language skills.

Comparison 4: Females comparing Low and High Language Skills

Since the p-value for β_{12} is 0.4370, which is larger than $\frac{\alpha}{4} = 0.0125$, we can conclude that there is no evidence of a difference in the rate at which math ability affects attendance for females with low language skills versus females with high language skills.

In order to understand the effect of gender on the relationhip between math ability and attendance, we need to look at the confidence intervals for the relevant parameters. We will first consider the effect for the students with high language ability:

The relevant parameter represents interaction between math ability and gender when language ability is centered at the higher value. The confidence interval for this parameter is 0.0069 to 0.0256. After applying the exponential function to these values, we will be able to see that the effect for females (sex=1) is between 1.007 and 1.026 times as great as for males (sex=0). This means that the effect is 0.7% to 2.6% greater for females than for males.

Next we will consider the effect for the students with low language ability:

The relevant parameter represents interaction between math ability and gender when the language ability is centered at the lower value. The confidence interval for this parameter is 0.0130 to 0.0280. After applying the exponential function to these values, we will be able to see that the effect for females is between 1.013 and 1.028 times as great as for males. This means that the effect is 1.3% to 2.8% greater for females than for males.

Overall language ability has no statistically significant effect on the way in which gender impacts the relationship between math ability and attendance. However, there is evidence of gender difference in the impace of language ability on the relationship between math ability and attendance.

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[2] Jeremy F. Dawson, *Moderation in Management Research: What, Why, When, and How*, J Bus Psychol Journal of Business and Psychology **29** (2013), 1–19.

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APPENDIX A

LETTER FROM INSTITUTIONAL RESEARCH BOARD

MARSHALL UNIVERSITY®

www.marshall.edu

Office of Research Integrity

February 8, 2016

Melinda Bierhals
5724 Stiles Drive, Apt 12
Huntington, WV 25705

Dear Ms. Bierhals:

This letter is in response to the submitted thesis abstract to produce confidence intervals that compare rates in Poisson regression with 3-way interaction. After assessing the abstract it has been deemed not to be human subject research and therefore exempt from oversight of the Marshall University Institutional Review Board (IRB). The Code of Federal Regulations (45CFR46) has set forth the criteria utilized in making this determination. Since the information in this study does not involve human subjects as defined in the above referenced instruction it is not considered human subject research. If there are any changes to the abstract you provided then you would need to resubmit that information to the Office of Research Integrity for review and a determination.

I appreciate your willingness to submit the abstract for determination. Please feel free to contact the Office of Research Integrity if you have any questions regarding future protocols that may require IRB review.

Sincerely,

Bruce F. Day, ThD, CIP
Director

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APPENDIX B

Analysis of Maximum Likelihood Parameter Estimates

X_3 = Sex - 0

Analysis Of Maximum Likelihood Parameter Estimates							
Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept	1	2.7464	0.1519	2.4488	3.0440	327.11	<.0001
x1	1	-0.0220	0.0046	-0.0309	-0.0130	23.31	<.0001
x2	1	-0.0128	0.0048	-0.0223	-0.0033	6.97	0.0083
x3	1	-0.1402	0.2233	-0.5778	0.2975	0.39	0.5302
x12	1	0.0002	0.0001	-0.0000	0.0004	3.25	0.0714
x13	1	0.0242	0.0062	0.0121	0.0363	15.43	<.0001
x23	1	-0.0061	0.0062	-0.0182	0.0061	0.96	0.3278
x123	1	-0.0001	0.0001	-0.0003	0.0001	0.97	0.3243
Scale	0	1.0000	0.0000	1.0000	1.0000		

Table B.1: Parameter Estimates for Males (Sex = 0)

X_3 = Sex - 1

Analysis Of Maximum Likelihood Parameter Estimates							
Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept	1	2.6063	0.1637	2.2853	2.9272	253.37	<.0001
x1	1	0.0022	0.0042	-0.0059	0.0104	0.29	0.5898
x2	1	-0.0188	0.0039	-0.0264	-0.0113	23.79	<.0001
x3	1	-0.1402	0.2233	-0.5778	0.2975	0.39	0.5302
x12	1	0.0001	0.0001	-0.0001	0.0002	0.60	0.4370
x13	1	0.0242	0.0062	0.0121	0.0363	15.43	<.0001
x23	1	-0.0061	0.0062	-0.0182	0.0061	0.96	0.3278
x123	1	-0.0001	0.0001	-0.0003	0.0001	0.97	0.3243
Scale	0	1.0000	0.0000	1.0000	1.0000		

Table B.2: Parameter Estimates for Females (Sex = 1)

X_2 = **Lanuguage - 32.1245876**

Analysis Of Maximum Likelihood Parameter Estimates							
Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept	1	2.3360	0.1033	2.1335	2.5386	510.93	<.0001
x1	1	-0.0165	0.0028	-0.0220	-0.0109	33.70	<.0001
x2	1	-0.0128	0.0048	-0.0223	-0.0033	6.97	0.0083
x3	1	-0.3348	0.1424	-0.6138	-0.0557	5.53	0.0187
x12	1	0.0002	0.0001	-0.0000	0.0004	3.25	0.0714
x13	1	0.0205	0.0038	0.0130	0.0280	28.51	<.0001
x23	1	-0.0061	0.0062	-0.0182	0.0061	0.96	0.3278
x123	1	-0.0001	0.0001	-0.0003	0.0001	0.97	0.3243
Scale	0	1.0000	0.0000	1.0000	1.0000		

Table B.3: Parameter Estimates for Language Low

X_2 = **Lanuguage - 68.0030042**

Analysis Of Maximum Likelihood Parameter Estimates							
Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr > ChiSq
Intercept	1	1.8776	0.2324	1.4222	2.3331	65.29	<.0001
x1	1	-0.0103	0.0040	-0.0181	-0.0025	6.70	0.0096
x2	1	-0.0128	0.0048	-0.0223	-0.0033	6.97	0.0083
x3	1	-0.5521	0.2841	-1.1090	0.0048	3.78	0.0520
x12	1	0.0002	0.0001	-0.0000	0.0004	3.25	0.0714
x13	1	0.0163	0.0048	0.0069	0.0256	11.65	0.0006
x23	1	-0.0061	0.0062	-0.0182	0.0061	0.96	0.3278
x123	1	-0.0001	0.0001	-0.0003	0.0001	0.97	0.3243
Scale	0	1.0000	0.0000	1.0000	1.0000		

Table B.4: Parameter Estimates for Language High

MELINDA BIERHALS

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EDUCATION

Marshall University, Huntington, West Virginia, USA ***Expected Graduation Date May 2016***
M.A., Mathematics

Bethany College, Bethany, West Virginia, USA ***May 2014***
B.S., Mathematics
Minor in Secondary Education

- *Cum Laude*
- Senior Project: *Personalities in Mathematics*: A study conducted on students to determine the possibilities of their personalities affecting their math grades.

ACADEMIC EXPERIENCE

Marshall University,Huntington, West Virginia, USA
Graduate Assistant ***August 2014 - Present***

- Primary Instructor for College Algebra Expanded, MTH 127
  January 2016 - Present
- Primary Instructor for Preparation for College Mathematics A, MTH 100
  August 2015 - December 2015
- Primary Instructor for Preparation for College Mathematics B, MTH 102
  August 2015 - December 2015
- Tutor Mathematical Courses through 300-level classes **August 2014 - Present**
- Primary Instructor for Mathematical Skills I, MTH 098 **August 2014 - May 2015**

Bethany College, Bethany, West Virginia, USA

Department of Physical and Computational Sciences

- Mathematics Fellow ***August 2013 - May 2014***
- Mathematics Tutoring Coordinator ***August 2013 - May 2014***
- Grade papers for College Algebra and Calculus I ***August 2010 - May 2014***
- Mathematics tutor ***August 2010 - May 2014***
  (All courses through the calculus series and discrete mathematics)

Serve as Mathematics Room Coordinator for the Department of Education ***March 2013- May 2014***
Serve as Math Science Day Coordinator ***August 2012 - March 2014***

PROFESSIONAL EXPERIENCE

Marshall University, Huntington, West Virginia, USA
Graduate Assistant ***August 2014 - Present***

Girl Scouts of Western Pennsylvania, Camp Redwing, Renfrew, Pennsylvania, USA
Unit Leader ***May 2015 - August 2015***

National Aeronautics and Space Administration (NASA) Goddard Space Flight Center, Greenbelt, Maryland, USA
Information Technology & Communication Directorate Financial Risk Assessment Intern
Summer 2014

Brooke High School, Wellsburg, West Virginia, USA
Student Teacher ***October 2013 - November 2013***

Wellsburg Middle School, Wellsburg, West Virginia, USA
Student Teacher ***August 2013 - October 2013***

Girl Scouts of Western Pennsylvania, Camp Redwing, Renfrew, Pennsylvania, USA
Camp Counselor ***June 2013 - August 2013***

II-VI Incorporated, Saxonburg, Pennsylvania, USA
Product Inventory Control Personnel ***June 2012 - August 2012***

CLINICAL EXPERIENCE

Bridge Street Middle School, Wheeling, West Virginia, USA
Classroom Discipline Observation/Practicum 10 hours ***Spring 2013***

Brooke High School, Wellsburg, West Virginia, USA
Methods and Materials in Teaching Observation/Practicum 30 hours ***Spring 2013***

Garfield Elementary School, Steubenville, Ohio, USA
The Special Education Process Observation/Practicum 20 hours ***Spring 2013***

Wellsburg Middle School, Wellsburg, West Virginia, USA
Middle School Methods Observation/Practicum 15 hours ***Sring 2013***

Wellsburg Middle School, Wellsburg, West Virginia, USA
Content Area Literacy Observation/Practicum 10 hours ***Fall 2012***

Bridge Street Middle School, Wheeling, West Virginia, USA
Professional Principles Observation 10 hours ***Fall 2011***

Garfield Elementary School, Steubenville, Ohio, USA
Exceptionalities and Diversities Observation 10 hours ***Fall 2011***

Holy Sepulcher Catholic School, Butler, Pennsylvania, USA
Multi-Cultural Field Experience Observation 20 hours ***Fall 2011***

RESEARCH EXPERIENCE

NASA Goddard Space Flight Center, Greenbelt, Maryland, USA
Research Advisor: Robert Spector
Historic Risk Data Collection and Analysis Effort ***Summer 2014***

PROFESSIONAL AND SOCIAL MEMBERSHIPS

American Mathematical Society, Providence, Rhode Island ***November 2015 - Present***
Professional Mathematics Society
Member

West Virginia Education Association, Charleston, West Virginia, USA ***August 2013 - Present***
Education Employee Organization
Member

Girl Scouts of Western Pennsyvania, Pennsyvania, USA ***June 2013 - Present***
Adult Membership

West Virginia Council of Teachers of Mathematics, West Virginia, USA

Professional mathemematics education group ***March 2013 - Present***
Member

Alpha Phi Omega, Bethany, West Virginia, USA ***2012 - 2014***
National Service Fraternity
Alpha Epsilon Chi (Bethany College) Chapter
Chapter President **January 2013 - May 2014**
Alumna

Kappa Delta Pi, Indianapolis, Indiana, USA ***2012 - 2014***
International Honor Society in Education
West Virginia Alpha Beta Tau (Bethany College) Chapter
Alumna

Kappa Mu Epsilon, Warrensburg, Missouri, USA, ***2012 - 2014***
National Mathematics Honorary Society
West Virginia Alpha (Bethany college) Chapter
President **February 2013 - February 2014**
Alumna

Students National Education Association, West Virginia, USA ***2012- 2014***
Professional Employee Organization
Member

Bethany Math and Computer Science Club, Bethany, West Virginia, USA ***2010 - 2014***
President **March 2013 - March 2014**
Secretary and Treasurer **March 2012 - March 2014**
Member

Bethany College Softball Club, Bethany, West Virginia, USA ***August 2010 - August 2013***
President **February 2013 - February 2014**
Member

TRAINING SESSIONS ATTENDED

Hawkes Courseware ***19 August 2015***

CPR Certification ***1 June 2015***

Lifeguard Certification ***1 June 2015***

Information Technology Security Training ***26 June 2014***

Goddard Space Flight Center Hazard Communication Training Program ***26 June 2014***

"Safe" Physical Intervention Support Techniques Training	***22 August 2013***
Child Abuse and Neglect Identification Training	***21 August 2013***
Kalon Leadership Acadamey	***23 February 2013***
Chromebook Workshop	***27 November 2012***
21^{st} Century Content Workshop	***17 October 2012***
I-pads in the Classroom	***10 Semptember 2012***

TECHNOLOGY FAMILIARITY

- Educreations (Flipped Classroom Application)
- Elmo
- Geometers Sketchpad
- Hawkes Courseware
- Ipad Technology
- JAVA
- LaTeX
- Mathematica
- Microsoft Office Programs
- Prezi
- Statistical Analysis System (SAS)
- Schoology
- Show Me (Flipped Classroom Application)
- Smartboard Technology
- Think Through Math
- TI-Nspire
- TI-84

PRESENTATIONS GIVEN

Miami University of Ohio Annual Math Conference 2014, Miami University, Oxford, Ohio, USA

Personalities In Mathematics — ***September 2014***
There are many aspects of a student that can affect how they learn. This research project considers a students' personality to be one of those aspects. College students were tested to find their personality type and those were compared to the grades they received in the class.

NASA Intern Poster Session, NASA Goddard Space Flight Center, Greenbelt, Maryland, USA

Historic Risk Data Collection and Analysis Effort — ***August 2014***
Report on Information Technology & Communications Directorate Financial Risk Assessment Research Project.

Bethany College Student Teacher Program, Bethany, West Virginia, USA

What is a Philosophy Statment and how is One Written? ***April 2014***
A facilitated open discussion for student teachers about the purpose of a teaching philosophy statemtent, and tips on how to write an effective one.

West Virginia Council of Teachers of Mathematics Conference, Stonewall Resort, West Virginia, USA

Personalities in the Classroom ***March 2014***
Session Description: Is there a correlation betwwen a students' personality and the grade they receive in a math class? This session is a continuation from Personalities in Mathematics. The results of personality tests that were taken by students will be compared to the grades those students earned in math classes. There are many aspects of a student that can affect their learning and personalities could be one of them.

West Virginia Council of Teachers of Mathematics Conference, Stonewall Resort, West Virginia, USA

Personalities in Mathematics ***March 2013***
Session Description: This session will explore the different personalities that people can have in math classes. Every person has a different type of personality and that may or may not have an effect on what types of grades they will receive in their math classes. There are many different aspects of a student that can affect their grades in math and their personality could be one of them.

PUBLICATIONS

"Historical Risk Data Collection and Analysis Effort", Melinda Bierhals. *NASA West Virginia Space Grant Consortium 2014-2015 Research Reports*, 2015, pp. 12 - 17.

AWARDS AND HONORS RECEIVED

Camp Redwing, Renfrew, Pennsylvania, USA

Received 2015 Staff of the Year Award in Serving Campers, Staff, and Parents. ***August 2015***

Bethany College, Bethany, West Virginia, USA

Received James E. Allison Award for Excellence in Teaching Mathematics ***April 2014***

Received Bethany College Fellow Recognition ***April 2014***

Departmental Fellow for Department of Physical and Computational Sciences

August 2013 - May 2014

Presidents Athletic Conference Spring 2013 Academic Honor Roll ***May 2013***

Received National Fast-Pitch Coaches Association Award All-American Scholar Athlete

May 2013

Received Distinguished Dispositions Inventory for Special Education 207 ***March 2012***

Presidents Athletic Conference Spring 2012 Academic Honor Roll ***May 2012***

Received National Fast-Pitch Coaches Association Award All-American Scholar Athlete

May 2012

Presidents Athletic Conference Spring 2011 Academic Honor Roll ***May 2011***

Received National Fast-Pitch Coaches Association Award All-American Scholoar Athlete

May 2011

CONFERENCES AND CONVENTIONS ATTENDED

West Virginia Council of Teachers of Mathematics Conference, Stonewall Resort, West Virginia, USA

Hosted by: West Virginia Council of Teachers of Mathematics ***March 2015***

Women and Technology Conference, Olgebay Resort, Wheeling, West Virginia, USA

Hosted by: TechConnect West Virginia ***October 2014***

Miami University of Ohio Annual Math Converence, Oxford, Ohio, USA

Hosted by: Miami University of Ohio ***September 2014***

Kappa Mu Epsilon National Convention, Jacksonville, Alabama, USA

Hosted by: Alabama Theta Chapter, Jacksonville University ***April 2014***

West Virginia Council of Teachers of Mathematics Conference, Stonewall Resort, West Virginia, USA

Hosted by: West Virginia Council of Teachers of Mathematics ***March 2014***

2013 Midstates Conference for Undergraduate Research, Delaware, Ohio, USA

Funded by: National Science Foundation
Hosted by: Ohio Wesleyan University Department of Mathematics and Computer Science

November 2013

West Virginia Council of Teachers of Mathematics Conference, Stonewall Resort, West Virginia, USA

Hosted by: West Virginia Council of Teachers of Mathematics ***March 2013***

2012 Midstates Converence for Undergraduate Research, Delaware, Ohio, USA

Funded by: National Science Foundation
Hosted by: Ohio Wesleyan University Departement of Mathematics and Computer Science

November 2012

REFERENCES

Available Upon Request