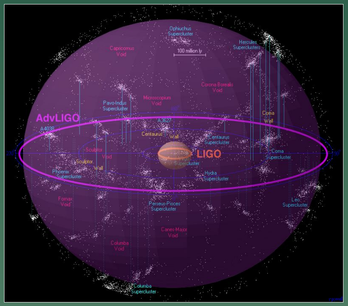
220th AAS Meeting – Anchorage, AK – June 10-14, 2012

Gravitational-Waveform Extraction by the Characteristic Method

Maria Babiuc-Hamilton
Department of Physics, Marshall University

Abstract

- When a pair of black holes spiral into each other and collide, the very fabric of space-time shakes, and gravitational waves are created.
- Gravitational waves carry information about their source, and will increase our understanding of relativistic systems in astrophysics.
- Gravitational wave observatories like *LIGO* and *Virgo* are tuned to detect the emission of these waves from the inspiral and merger of binary black holes, neutron starts, supernovae, etc…
- **Problem**: any small vibration is detected, so *templates* are essential to discern the real signal.
- It is hard to compute the waveforms obtained from numerical simulations accurately – gravitational radiation is properly defined only at *null infinity*, but is estimated at a finite radius.
- **Cauchy-Characteristic Extraction (CCE)** is the most precise and refined "extraction" method available. The **CCE** technique connects the strong-field "Cauchy" evolution of the space-time near the merger to the "characteristic" evolution far from the merger – at null infinity, where the waveform is extracted and detectors measure it.
- We present a stand-alone "characteristic" waveform extraction tool that has demonstrated accuracy and convergence of the numerical error and is used by the numerical relativity groups for the unambiguous extraction of waveforms.
- We prove that the numerical error of **CCE** satisfies the standards of the detection criteria required for Advanced LIGO data analysis.
- The tool provides a means for accurate calculation of waveforms generated by evolution codes based upon different analytic and numerical formulations of the Einstein equations.



Anticipated reach of Advanced LIGO

Formalism

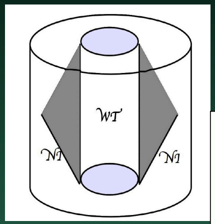
- Cauchy-characteristic method covers all space-time by combining 2 regions
  1. A timelike (Cauchy) close to BBH
  2. A null (characteristic) far field.
- The characteristic evolution is embedded in the Cauchy evolution
- Cauchy + characteristic initial-values

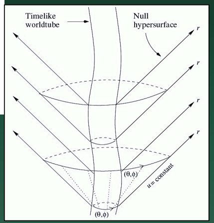
Cauchy Initial data:

- Close quasicircular black hole binary inspiral with orbital frequency *0.05*.

Characteristic initial Data

- Weyl condition on the initial null hypersurface to suppress incoming radiation, vanishes at infinity.





- The data is decomposed in Chebyshev and spherical harmonics coefficients on a band *R±dr*.
- Then is reconstructed in characteristic Bondi-Sachs coordinates, and evolved on the light cones
- Infinity is included in the computational grid by Penrose compactification of the radial coordinate:

$$x = \frac{r}{R_E + r}$$

- Einstein equations $G_{\mu\nu}=0$ evolved radially outward in Bondi-Sachs coordinates

$$ds^2 = -\left(e^{2\beta}\frac{V}{r} - r^2 h_{AB}U^A U^B\right)du^2 - 2e^{2\beta}dudr - 2r^2 h_{AB}U^B du dx^A + r^2 h_{AB}dx^A dx^B$$

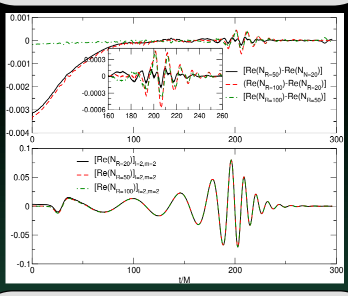
- Waveforms computed at null infinity in conformal compactified Bondi coordinates *l=1/r*

$$d\hat{s} = -\left(e^{2\beta}Vl^3 - h_{AB}U^A U^B\right)du^2 + 2e^{2\beta}dudl - 2h_{AB}U^B du dx^A + h_{AB}dx^A dx^B$$

Waveforms

- The waveform is extracted in terms of the Bondi News *N* and the Weyl tensor Ψ_4

$$N = N_+ + iN_\times = \partial_t h_+ + \partial_t h_\times,\ \ \Psi_4 = l\partial_u N + O(l^2)$$



- Plots of the dominant(*l=2, m=2*) mode of Richardson extrapolated waveform $N_E(t)$ obtained with extraction radii R_E*=20M, 50M*, and *100M*.
- The R_E*=50M* and R_E*=100M* waveforms are shifted backward in time to be in phase at the peak.
- Two sources of "junk" radiation:
  ① Choice of conformally flat initial Cauchy data
  ② Initial Cauchy and characteristic data mismatch
- The three waveforms are in good agreement in the inspiral and merger stage, with relative difference between the R_E=20M and R_E*=100M* is *0.6%* .

Advanced Accuracy Standards

- Sensibility of detector, given in frequency domain
- Translated into the time domain, the error of a numerical waveform *for strain, news and* Ψ_4 comp:

$$\varepsilon_0 = \|\delta h\|/\|h\|,\ \varepsilon_1 = \|\delta N\|/\|N\|,\ \varepsilon_2 = \|\delta\Psi_4\|/\|\Psi_4\|$$

- Criteria for waveform accuracy
  ① Accuracy for detection

$$\varepsilon_k \le C_k\sqrt{2\varepsilon_{max}};\ \varepsilon_{max} - template\ mismatch$$

  ② Accuracy for measurement

$$\varepsilon_k \le C_k\frac{\eta_c}{\rho};\ \eta_c - detector\ calibration$$

- Required accuracy or detection:

$\varepsilon_{max} = 0.005,\ 0.24 \le C_1 \le 0.8 \Rightarrow \varepsilon_1 \le 0.1C_1 \le 0.024$

- Required accuracy for measurement:

$h_{min} = 0.4,\ C_1 = 0.24,\ r=100 \Rightarrow \varepsilon_1 \le 9.6\times10^{-2}/r \le 9.6\times10^{-4}$

Requirements for the Bondi News

1. The criterion for detection <u>is satisfied</u> *throughout the entire binary mass range* and is unaffected by choice of extraction radius.
2. The criterion for measurement <u>is also satisfied</u> *throughout the entire binary mass range.*

Variable	*Re*	*Im*
$\mathcal{E}_1(N)_{R=20}$	8.76×10^{-4}	8.74×10^{-4}
$\mathcal{E}_1(N)_{R=50}$	2.62×10^{-4}	2.60×10^{-4}
$\mathcal{E}_1(N)_{R=100}$	1.21×10^{-4}	1.22×10^{-4}
$\mathcal{E}_1(N_{\Delta R(20,100)})$	5.41×10^{-3}	5.55×10^{-3}
$\mathcal{E}_1(N_{\Delta R(50,100)})$	4.28×10^{-3}	4.51×10^{-3}

Requirements for the Weyl Tensor

1. The criterion for detection <u>is satisfied</u> *throughout the entire binary mass range* in the high mass limit.
2. The values at all three extraction radii satisfy the measurement requirement for advanced LIGO signal-to-noise ratio $\rho = 100$.

Variable	*Re*	*Im*
$\mathcal{E}_2(\Psi)_{R=20}$	1.138×10^{-3}	1.174×10^{-3}
$\mathcal{E}_2(\Psi)_{R=50}$	4.038×10^{-4}	3.531×10^{-4}
$\mathcal{E}_2(\Psi)_{R=100}$	2.810×10^{-4}	2.093×10^{-4}
$\mathcal{E}_2(\Psi_{\Delta R(20,100)})$	5.391×10^{-2}	4.148×10^{-2}
$\mathcal{E}_2(\Psi_{\Delta R(50,100)})$	1.937×10^{-2}	1.905×10^{-2}

Conclusions

- The aim of **CCE** is to provide a standardized gravitational waveform extraction tool.
- The new extraction tool contains major improvements and corrections to previous versions and displays convergence.
- The error introduced by **CCE** satisfies the time domain criteria required for advanced *LIGO* data analysis.
- The importance of accurate waveforms to the gravitational wave astronomy has created an urgency for tools like **CCE**.
- The source code has been released to the public and is available as part of the Einstein Toolkit.
- We welcome applications to a variety of generic numeric codes implementing Einstein Equations of General Relativity.