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Adding Light to the Gravitational Waves on the Null Cone

Maria Babiuc-Hamilton
Marshall University, babiuc@marshall.edu

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ADDING LIGHT TO THE GRAVITATIONAL WAVES ON THE NULL CONE

Maria Babiuc
Marshall University, WV

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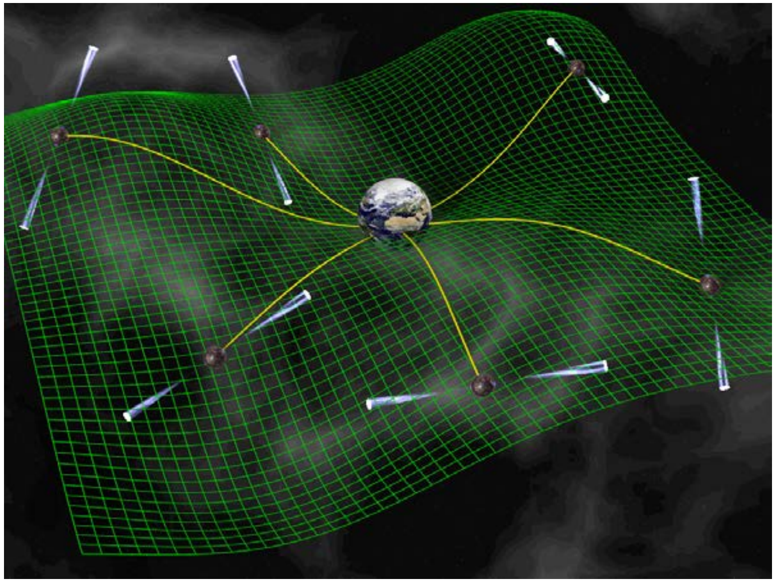
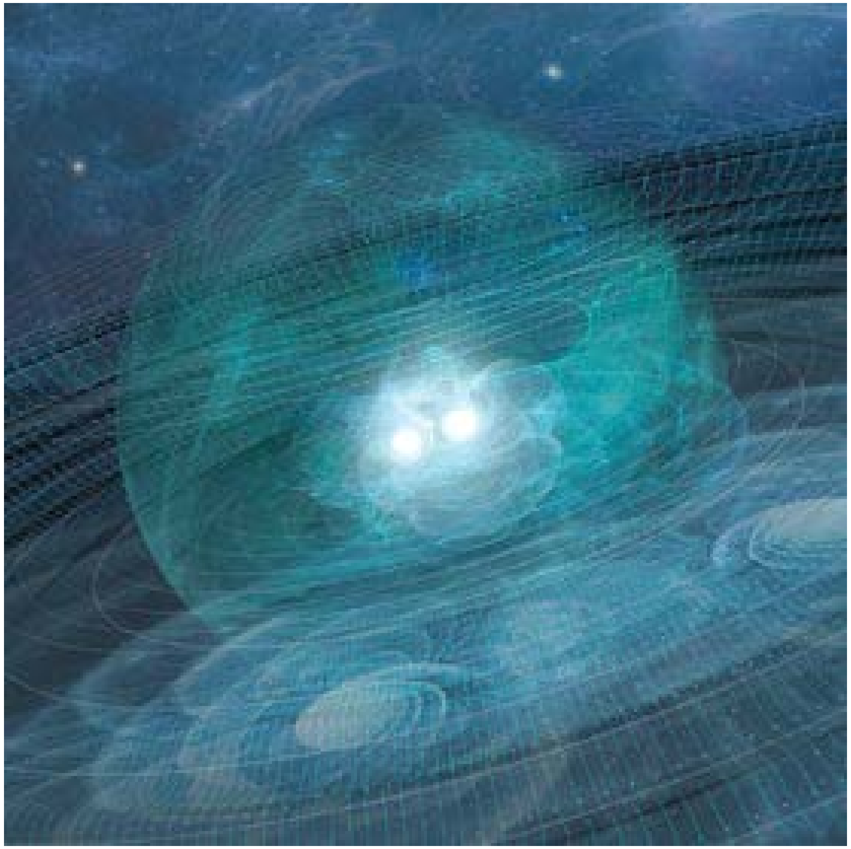
PTA and the search for GW



- The Pulsar Timing Array (PTA) will be able to detect not only stochastic GW, but also resolve individual super massive binaries through their EM emission.
- SMBH binaries reside in recent galaxies mergers, of massive, nearby host galaxies, with possible AGN.

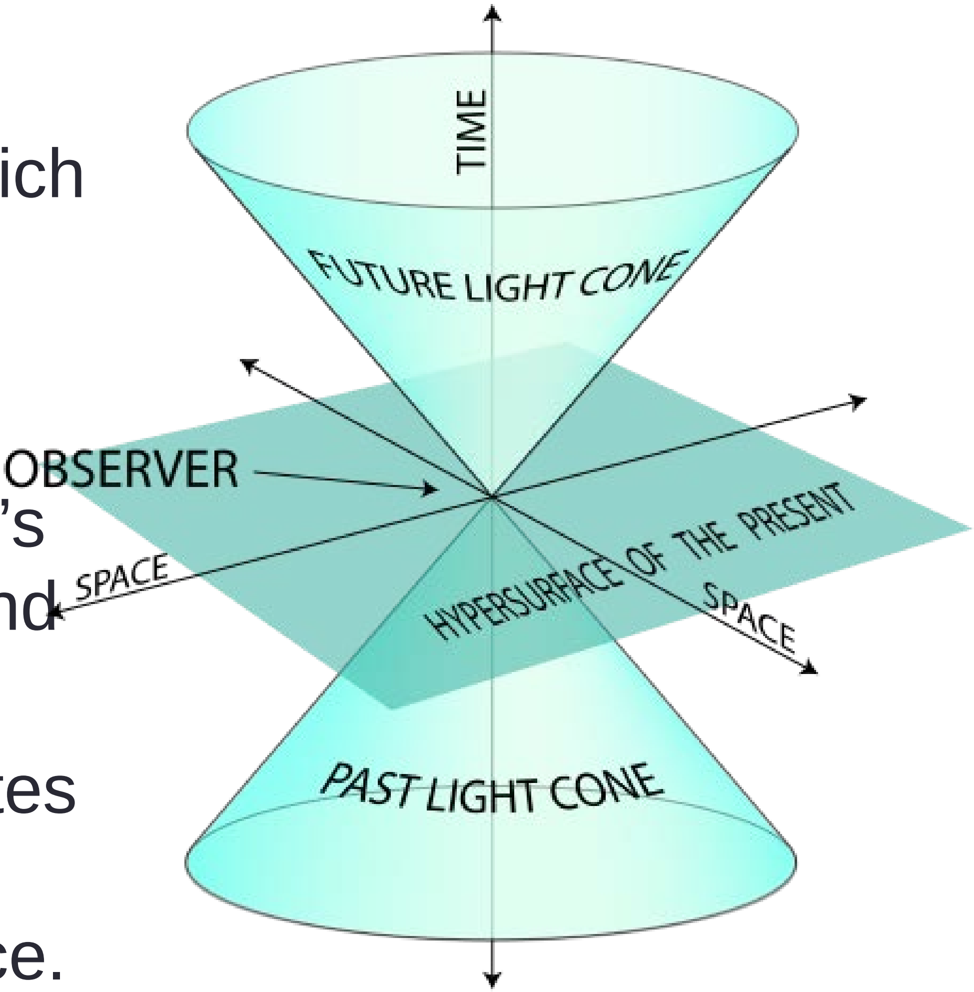
We Can See Gravitational Waves

- If the merger of two SMBHs produce a light signal.
- The accretion disks around merging BHs, emit EM radiation that includes the GW signature of the binary.
- Periodic flares on a timescale correlated with the GW frequency, or even the birth of a bright variable quasar, could indicate the presence of SMBH binaries.



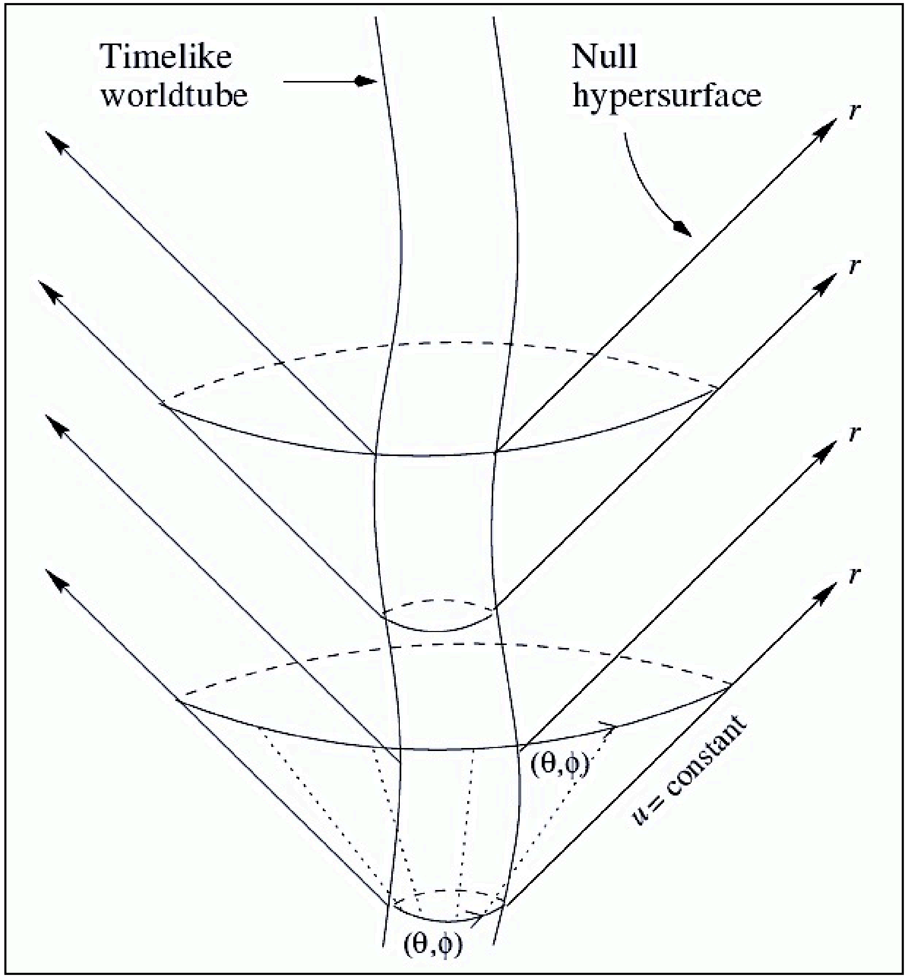
How Radiation Travels

- Both gravitational and electromagnetic radiation travel along light rays, which are ***principal null directions*** in space-time.
- They are ***characteristic surfaces*** both of Einstein's vacuum field equations and of Maxwell's equations.
- In characteristic coordinates Einstein equations split in evolution and hypersurface.



The Characteristic Code

- Einstein equations $G_{\mu\nu}=0$ are propagated radially and in time along the outgoing light rays, in ***Bondi-Sachs*** coordinates, by a marching integration algorithm.
- The gravitational waveforms are computed at positive null infinity on inertial Bondi coordinates in terms of the Bondi news and Weyl scalar.



Enlighten the Gravity

- Cover the whole space-time with the Sachs metric:

$$ds^2 = \left(-e^{2\beta}\frac{V}{r} + r^2U^2e^{2\psi} + r^2W^2e^{-2\psi}\right)du^2 - 2e^{2\beta}dudr - 2r^2We^{2\psi}dud\theta$$

$$-2r^2Y\sin\theta e^{-2\psi}dud\varphi + r^2(e^{2\psi}d\theta^2 + e^{-2\psi}\sin^2\theta d\varphi^2)$$

- Consider an EM field described by the Faraday tensor:

$$F = F_{01}du\wedge dr + F_{02}du\wedge d\theta + F_{03}du\wedge d\varphi + F_{12}dr\wedge d\theta + F_{13}dr\wedge d\varphi + F_{23}d\theta\wedge d\varphi$$

- Write down the field equations to be solved:

$$G_{\alpha\beta} = 8\pi T_{\alpha\beta}; \quad D_{[\delta}F_{\alpha\beta]} = 0; \quad D^{\alpha}F_{\alpha\beta} = 0$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \tfrac{1}{2}g_{\alpha\beta}R; \quad T_{\alpha\beta} = \tfrac{1}{4\pi}\left(F_{\alpha}^{\delta}F_{\beta\delta} - \tfrac{1}{4}g_{\alpha\beta}F_{\delta\sigma}F^{\delta\sigma}\right)$$

$$D^{\alpha}G_{\alpha\beta} = 0; \quad D^{\alpha}T_{\alpha\beta} = 0; \quad D^{\alpha}W_{\alpha\beta\delta\sigma} = \tfrac{1}{2}\left(D_{\delta}R_{\beta\sigma} - D_{\sigma}R_{\beta\delta}\right)$$

Make Reasonable Assumptions

- **Causality**: no inflow radiation from future.
- **Vacuum**: the Ricci scalar curvature is zero.
- **Axial symmetry** will not restrict the generality.
- **Linearity**: The parallax, and luminosity distance ***r*** along the rays agree.
- **Euclidean topology**: the space-time is flat at a distance reasonably far away from the source.

$$e^{\beta} \cong 1; V \cong r - 2\mathrm{M}(u,\theta)$$

$$\psi \cong \tfrac{1}{r}\Psi(u,\theta); U \cong \tfrac{1}{r^2}\Upsilon(u,\theta);$$

$$W \cong 0; F_{12} \cong 0; F_{03} \cong 0$$

$$F_{01} \cong \tfrac{1}{r^2}\mathrm{E}(u,\theta); F_{13} \cong \tfrac{1}{r^2}\mathrm{H}(u,\theta)$$

$$F_{02} \cong \mathrm{X}(u,\theta); F_{23} \cong \mathrm{Z}(u,\theta)$$

Book Keeping

- The linearized Sachs metric with axial symmetry is:

$$ds^2 = \left(-1 + \frac{2\mathrm{M}}{r} + \Upsilon^2 e^{\frac{2}{r}\Psi}\right) du^2 - 2dudr - 2r^2 \Upsilon e^{\frac{2}{r}\Psi} dud\theta + r^2 (e^{\frac{2}{r}\Psi} d\theta^2 + e^{-\frac{2}{r}\Psi} \sin^2\theta d\varphi^2)$$

- The Faraday tensor for the electromagnetic field is:

$$F = F_{01} du \wedge dr + F_{02} du \wedge d\theta + F_{13} dr \wedge d\varphi + F_{23} d\theta \wedge d\varphi$$

- The field equations split into twelve main equations:

$$R_{01} - 8\pi T_{01} = 0; \quad R_{12} - 8\pi T_{12} = 0; \quad R_{13} - 8\pi T_{13} = 0$$

$$R_{22} - 8\pi T_{22} = 0; \quad R_{23} - 8\pi T_{22} = 0; \quad R_{33} - 8\pi T_{33} = 0$$

$$D_{[2} F_{01]} = 0; \quad D_{[3} F_{01]} = 0; \quad D_{[3} F_{12]} = 0$$

$$D^{\alpha} F_{\alpha 0} = 0; \quad D^{\alpha} F_{\alpha 2} = 0; \quad D^{\alpha} F_{\alpha 3} = 0$$

- The unknown variables are: M, Y, Ψ, E, H, Z, X

The Questions are:

1. Can we still split all these equations into evolution and hypersurface equations for the main variables?
2. Can we decouple the gravitational and the EM fields?

- Ideally: with initial data for the gravitational field Ψ_0, and for the electromagnetic field ($\mathbf{E}_0$, $\mathbf{H}_0$), we calculate ($\mathbf{Y}_0$, $\mathbf{M}_0$, $\mathbf{Z}_0$, $\mathbf{X}_0$) and integrate to (Ψ_1, $\mathbf{E}_1$, $\mathbf{H}_1$).
- A hierarchical integration uses hypersurface equations for ($\mathbf{Y}$, $\mathbf{M}$, $\mathbf{Z}$, $\mathbf{X}$) and evolution equations for (Ψ, $\mathbf{E}$, $\mathbf{H}$).
- Hierarchical equations:
  1. $R_{12}-8\pi T_{12}=0 \Rightarrow Y_0$ ***function of*** Z_0
  2. $R_{22}=8\pi T_{12}=0 \Rightarrow M_0$ ***function of*** Z_0
  3. $R_{33}=8\pi T_{33}=0 \Rightarrow Z_0 \Rightarrow Y_0, M_0$
  4. $J_2=0 \Rightarrow X_0$
- Evolution equations: $R_{01}=8\pi T_{01} \rightarrow \Psi_1$; $J_0=0 \rightarrow E_1$; $J_3=0 \rightarrow H_1$

Numerical Algorithm

- The hierarchical equations contain only angular derivatives and will be integrated in Fourier space.
- The evolution equations can be integrated with a generic 4th order (Runge-Kutta) integrator, because the dependence on the extraction radius ***r*** is explicit.
- Construct the Weyl tensor, taking into account the EM

$$W_{\alpha\beta\delta\sigma} = R_{\alpha\beta\delta\sigma} - \frac{1}{2}\left(g_{\alpha\delta}E_{\beta\sigma} + g_{\beta\sigma}E_{\alpha\delta} - g_{\alpha\sigma}E_{\beta\delta} - g_{\beta\delta}E_{\alpha\sigma}\right)$$

$$E_{\alpha\beta} = R_{\alpha\beta} - 8\pi T_{\alpha\beta}$$

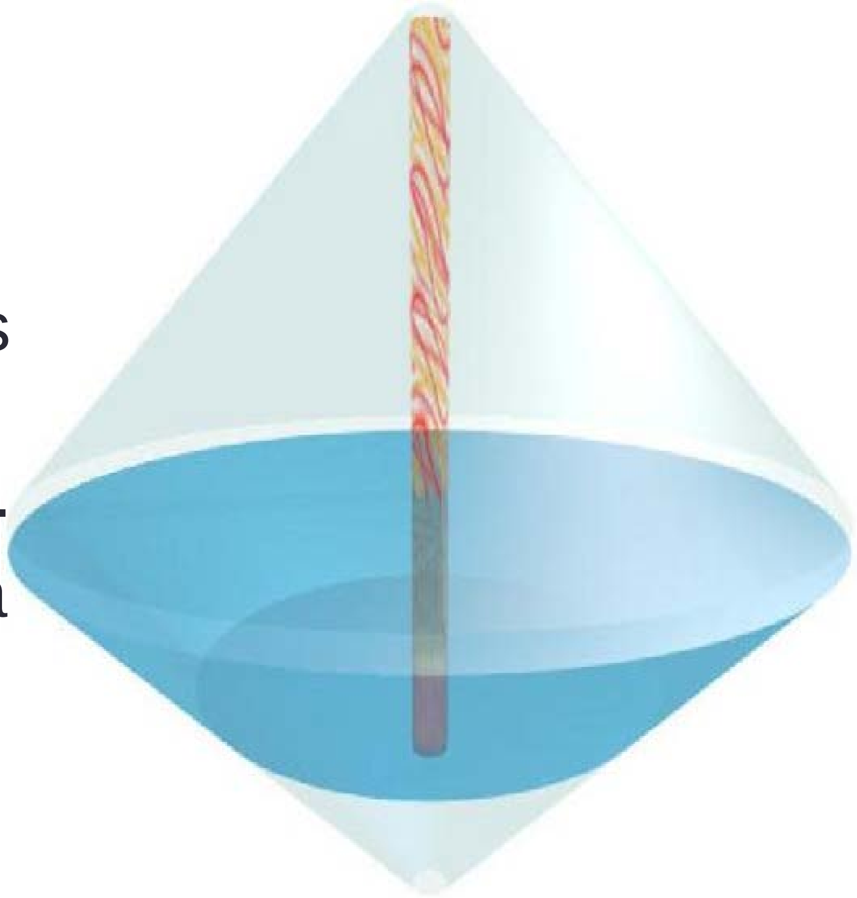
- Both the Weyl and Faraday fields will have to be projected onto a quasi-normal tetrad to extract the scalars encoding the measurable values of the fields.

What’s up?

- Future extensions of the code:
  - Drop the axial symmetry
  - Extend the power series expansion in ***r***
  - Add the radial dependence
  - Compactify the ***r*** coordinate
  - Change the integration algorithm
  - Carry the evolution to null infinity
  - Do the asymptotic expansion to a inertial coordinate system
  - Calculate the Weyl scalar and Poynting flux
  - Include null neutrino dust
  - Include universe expansion

A New Characteristic Code

- Numerical relativity groups could develop their own homemade characteristic extraction modules.
- Electromagnetic counterparts of gravitational waves can point to gravitational sources.
- Other interesting phenomena
  - gravitational memory effect
  - formation of trapped surfaces
  - the problem of horizon



The Gravitational Wave Spectrum