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Genetic Optimization of a Tensegrity Structure

Jaime R. Taylor

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GENETIC OPTIMIZATION OF A TENSEGRITY STRUCTURE

Prepared By:	Jaime R. Taylor
Academic Rank:	Associate Professor
Institution and Department:	Austin Peay State University Physics
NASA/MSFC Directorate:	Engineering
MSFC Colleague:	Jim Steincamp

Introduction

Marshall Space Flight Center (MSFC) is charged with developing advanced technologies for space telescopes. The next generation of space optics will be very large and lightweight. Tensegrity structures are built of compressive members (bars), and tensile members (strings). For most materials, the tensile strength of a longitudinal member is larger than its buckling strength; therefore a large stiffness to mass ratio can be achieved by increasing the use of tensile members[8]. Tensegrities are the epitome of lightweight structures, since they take advantage of the larger tensile strength of materials.

The compressive members of tensegrity structures are disjoint allowing compact storage of the structure. The structure has the potential to eliminate the requirement for assembly by man in space; it can be deployed by adjustments in its cable tension. A tensegrity structure can be more reliably modeled since none of the individual members experience bending moments. (Members that experience deformation in more than one dimension are much harder to model.) Structures that can be more precisely modeled can be more precisely controlled[8].

Furthermore, an astoundingly wide variety of natural systems, including carbon atoms, water molecules, proteins, viruses, cells, tissues and even human and other living creatures are tensegrity structures[2]. Through the process of evolution, nature continually improves the design of living creatures for the environment they live in. Since tensegrities are nature's structure of choice, it is conceivable that they have other benefits we are unaware of.

A. Keane and S. Brown designed a satellite boom truss system with an enhanced vibration performance. They started with a standard truss system, Figure 1, then used a genetic algorithm to alter the design, Figure 2, while optimizing the vibration performance. An improvement of over 20,000% in frequency-averaged energy levels was obtained using this approach[3].

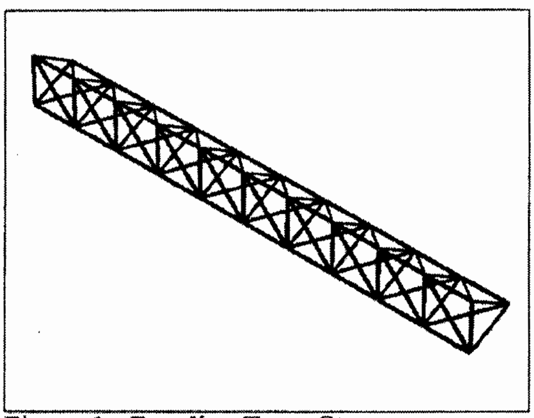

Figure 1. Baseline Truss Structure.

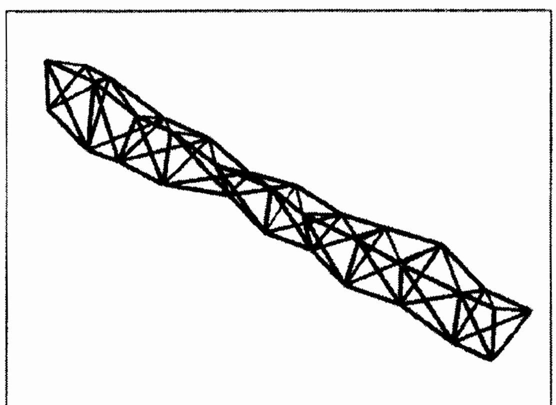

Figure 2. Performance Enhanced Structure.

In this report an introduction to tensegrity structures is given, along with a description of how to generate the nodal coordinates and connectivity of a multiple stage cylindrical tensegrity structure. A description of how finite elements can be used to develop a stiffness and mass matrix so that the modes of vibration can be determined from the eigenvalue problem is shown. A brief description of a micro genetic algorithm are then presented.

What is a Tensegrity Structure?

According to Pugh, a tensegrity (tension and integrity) structure consists of discontinuous compression members suspended by a continuous network of pure tension members all of which are pin-jointed[7]. Figure 3 shows a 2D tensegrity structure that fits this definition. C2T4 stands for 2 compressive members, bars, and 4 tension members, cables. Skelton defines Class k Tensegrity structures, where k is the maximum number of compressive members connected at the nodes. Figure 3 is a class 1 structure and Figure 4, a C4T2 structure, as a class 2 tensegrity structure[8].

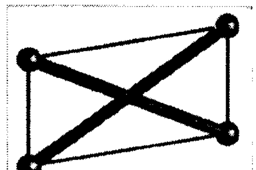
Figure 3. C2T4.

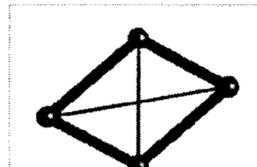
Figure 4. C4T2.

A detailed description on how to find the shape of a multi-stage cyclic-right-cylindrical-tensegrity is presented by Murakami and Nishimura[6]. Figure 5 shows a 4 stage, 6 bar tensegrity structure.

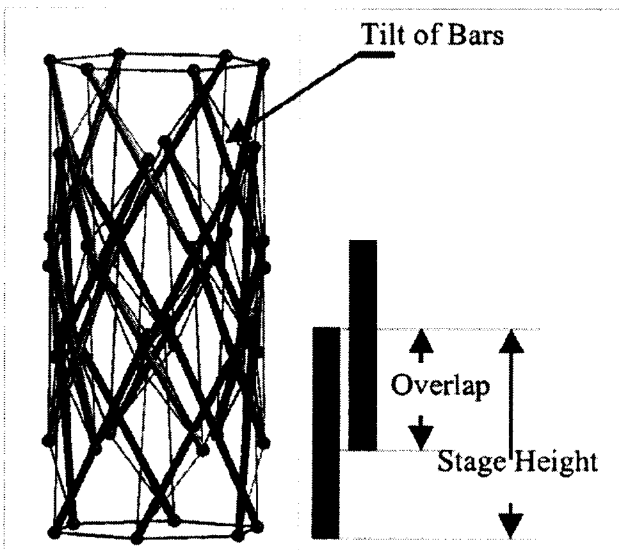

Figure 5. A 4 Stage, 6 Bar Tensegrity Structure.

A tensegrity structure can be modeled in the same fashion as a truss structure using finite elements. The difference between modeling a truss and tensegrity structure is that the tension members cannot undergo compression. Figure 6a shows the forces, ***F***, and displacements, ***u***, for a bar or tension member. A member, either bar or tension, is modeled as a spring with lumped masses on each end, Figure 6b. In Figure 6a *xy* are the global coordinates and $\overline{xy}$ are the local coordinates. The relationship between the local and global coordinates are given in Equation Set 1. {q} is the vector representation of the displacements $\{u_1, v_1, u_2, v_2\}$. [T] is the transformation matrix composed of $\cos(\phi)$ and $\sin(\phi)$[5].

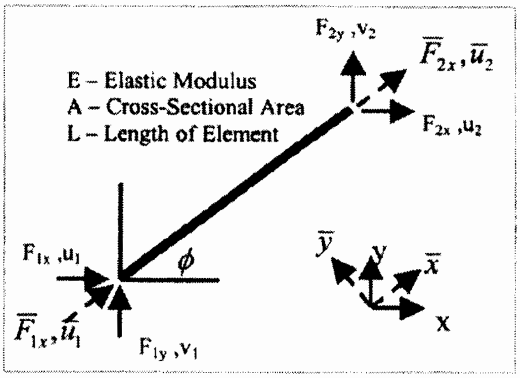

Figure 6a. Finite Element Model.

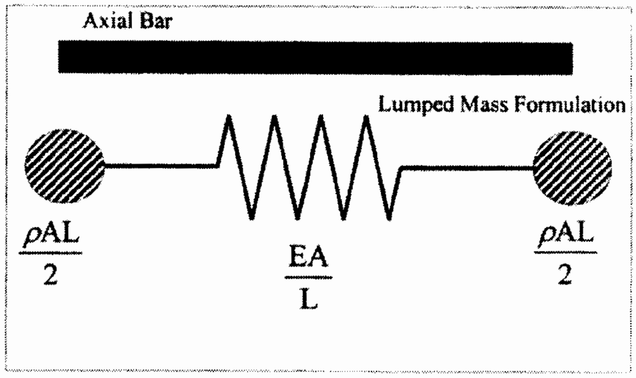

Figure 6b. Physical Model of Elements.

$$\begin{Bmatrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{u}_2 \\ \bar{v}_2 \end{Bmatrix} = \begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \qquad \begin{matrix} c = \cos(\phi) \\ s = \sin(\phi) \end{matrix} \qquad \begin{matrix} \{\bar{q}\} = [T]\{q\} \\ \{\bar{F}\} = [T]\{F\} \end{matrix} \qquad (1)$$

The stiffness matrix is given in Equation Set 2.

$$\begin{Bmatrix} \bar{F}_{1x} \\ \bar{F}_{1y} \\ \bar{F}_{2x} \\ \bar{F}_{2y} \end{Bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{u}_2 \\ \bar{v}_2 \end{Bmatrix} \qquad \begin{matrix} \{\bar{F}\} = [\bar{k}]\{\bar{q}\} \\ [k] = [T]^T[\bar{k}][T] \end{matrix} \qquad (2)$$

Once the global stiffness matrix is constructed for an individual element, all the stiffness matrices can be assembled into the total stiffness matrix for the structure. The global mass matrix is constructed in a similar fashion, as shown in the first two columns of Equation Set 3. The last column of equations shows the eigenvalue problem, where ω is the angular frequency of the modes of vibration of the structure.

$$[\bar{m}] = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \qquad \begin{matrix} [m] = [T]^T[\bar{m}][T]; \\ \{q\} = \{Q\}\sin\omega t; \end{matrix} \qquad \begin{matrix} \{F\} = [k]\{q\} + [m]\{\ddot{q}\} \\ \{F\} = ([k] - \omega^2[m])\{q\} \\ [0] = ([k] - \omega^2[m])\{q\} \end{matrix} \qquad (3)$$

Micro-Genetic-Algorithm

The goal is now to design a tensegrity structure while minimizing mass, increasing the lowest mode of vibration and maintaining the structural integrity of the tensegrity. The design parameters are n-number of stages, m-bars per stage, s-scale of pre-stress, the overlap between stages and the tilt of the bars (see Figure 5), and the radii of the top and bottom of each stage. A micro-genetic-algorithm is an ideal method for optimizing the structure[4]. (A classic book on genetic algorithm's is Goldberg[1].)

The micro-GA starts with a baseline solution similar to that for the truss system Figure 1. Four new individuals are randomly generated and centered about the baseline solution. The five individuals, four new ones and original "best of breed", are paired up for mating. The "Best" pair is chosen to mate and generate four new individuals; the "best of breed" is kept. This process is repeated, see last column in Figure 7, until the best fit no longer improves, a Restart is then performed. The flowchart for the micro-GA is given in Figure 7.

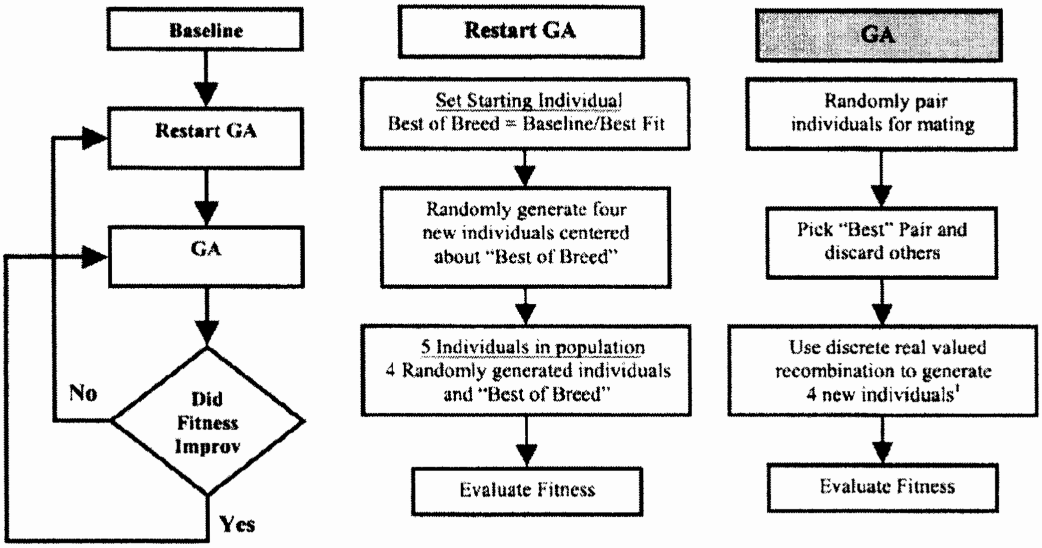

Figure 7. Flow Chart Describing a Micro-Genetic-Algorithm.

Acknoledgements

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