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Phase Retrieval Using a Genetic Algorithm on the Systematic Image-Based Optical Alignment Test Bed

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PHASE RETRIEVAL USING A GENETIC ALGORITHM ON THE SYSTEMATIC IMAGE-BASED OPTICAL ALIGNMENT TESTBED

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Introduction

NASA's Marshall Space Flight Center's Systematic Image-Based Optical Alignment (SIBOA) Testbed was developed to test phase retrieval algorithms and hardware techniques. Individuals working with the facility developed the idea of implementing phase retrieval by breaking the determination of the tip/tilt of each mirror apart from the piston motion (or translation) of each mirror. Presented in this report is an algorithm that determines the optimal phase correction associated only with the piston motion of the mirrors.

A description of the Phase Retrieval problem is first presented. The Systematic Image-Based Optical Alignment (SIBOA) Testbeb is then described. A Discrete Fourier Transform (DFT) is necessary to transfer the incoming wavefront (or estimate of phase error) into the spatial frequency domain to compare it with the image. A method for reducing the DFT to seven scalar/matrix multiplications is presented. A genetic algorithm is then used to search for the phase error. The results of this new algorithm on a test problem are presented.

What is Phase Retrieval?

Phase retrieval is the process of recovering wavefront error given a measurement of a Point-Spread-Function (PSF). An incoming wavefront has the form $Ae^{i\Phi}$ where A is the amplitude and Φ is the phase. A perfectly focused wavefront has zero phase error. The phase error cannot be measured directly, only the square of the amplitude of the Fourier transform (PSF). Figure 1a shows an uncorrected PSF and Figure 1b is the same PSF corrected.

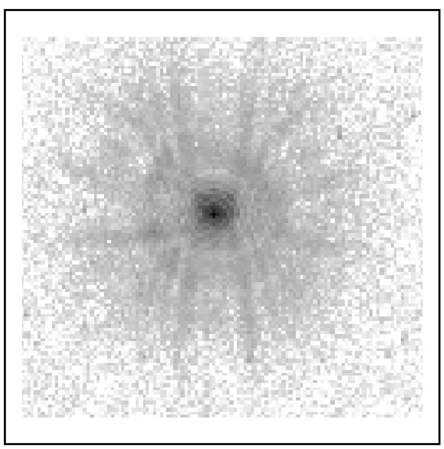

Figure 1a. Uncorrected PSF.

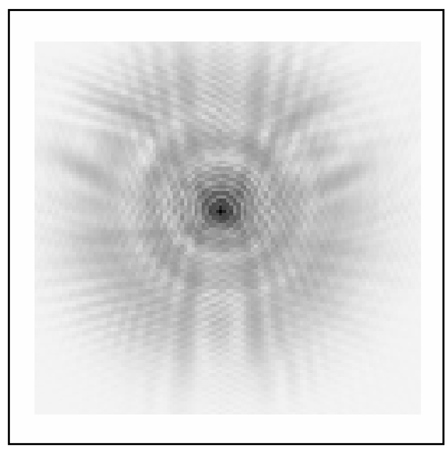

Figure 1b. Corrected PSF.

A known source is put through the optical system, the PSF is measured, and the phase distortion is calculated. Adaptive optics are then used to correct the phase [10]. Phase retrieval is much harder than standard image restoration due to the fact it is non-linear. Many solution techniques have been applied to the phase retrieval problem such as Iterative Fourier Transform Methods [4], optimization methods using simulated annealing [8], neural networks [9], and the simplex method [5], and other techniques such as the application of the wavelet transform [1]. Reference [6] and [7] present the phase retrieval problem with more mathematical rigor.

Major Problems that exist are unpredictability of convergence (iterative Fourier transform methods), converges to local minimum (maximum likelihood approaches tend to get stuck at local minima), and a vast number of necessary FFT calculations. This work addresses all these concerns.

SIBOA Testbed and Problem Statement

The Systematic Image-Based Optical Alignment (SIBOA) Testbed is a seven mirror adaptive optics system, see Figure 2 below. Each mirror has the ability to tip/tilt and to translate under piston motion. The individuals working with this system have implemented sensing hardware that allows the tip/tilt to be corrected independently of the piston motion. An algorithm already exists for the fast reliable solution of the tip/tilt problem [2]. The problem addressed here is finding the phase error to be corrected with the piston motion.

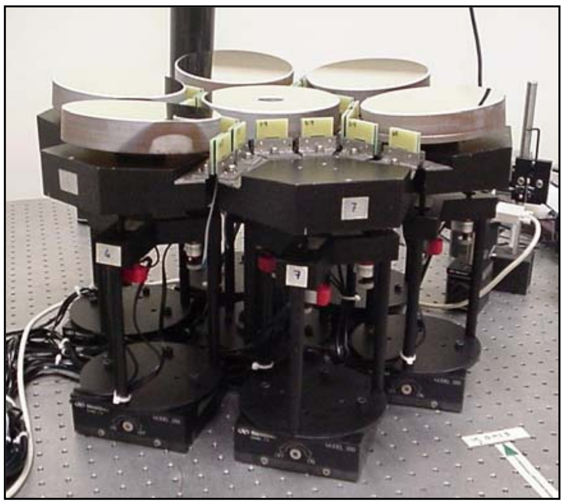

Figure 2. The Systematic Image-Based Optical Alignment Testbed.

Reduced Discrete Fourier Transform

It is possible to reduce the DFT, Equation 1, to seven scalar/matrix multiplications due to the fact the tip/tilt and the piston motion of the mirrors are sensed and corrected with independent hardware. The piston motion results in a uniform phase change across the surface of the mirror.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\left(2\frac{\pi}{N}\right)nk} \tag{1}$$

Due to this fact *x(n)* can be written as $re^{i\Phi}$ where *r,* the amplitude, is one at the mirror locations and zero outside the mirrors, and Φ is a constant value between $-\pi$ and π for each mirror. Since Φ is constant for each mirror the phase term, $e^{i\Phi}$, can be factored out of the calculation and the DFT for each mirror need only be calculated once for a particular correction cycle. Each time a new set of phase parameters is tested, the phase term for each mirror (a scalar) is multiplied by the DFT term for each mirror (a matrix). These terms are then added to produce the complete DFT.

Genetic Algorithm

A general description of binary GAs is provided in the classic text by David Goldberg [3]. For this application a binary GA was used with uniform crossover with reduced surrogate. After every ten generations the population was "restarted" using a new range, given by the min and max parameter values of the top forty individuals in the population, centered about the average of the top five individuals. Restarting the population did three things, it reintroduced new genetic material that might have been lost, it reduced the search space, and most importantly in increased the resolution within this reduced search space.

Results

The PSF was calculated using a known phase error. The algorithm developed here was applied to the PSF to estimate the known phase error; the estimated error was then compared to the known error. Several tests cases were examined. The GA was able to determine the phase error to whatever accuracy the user wanted up to machine precision. Figure 3 shows an example of the evolution of the populations in the GA.

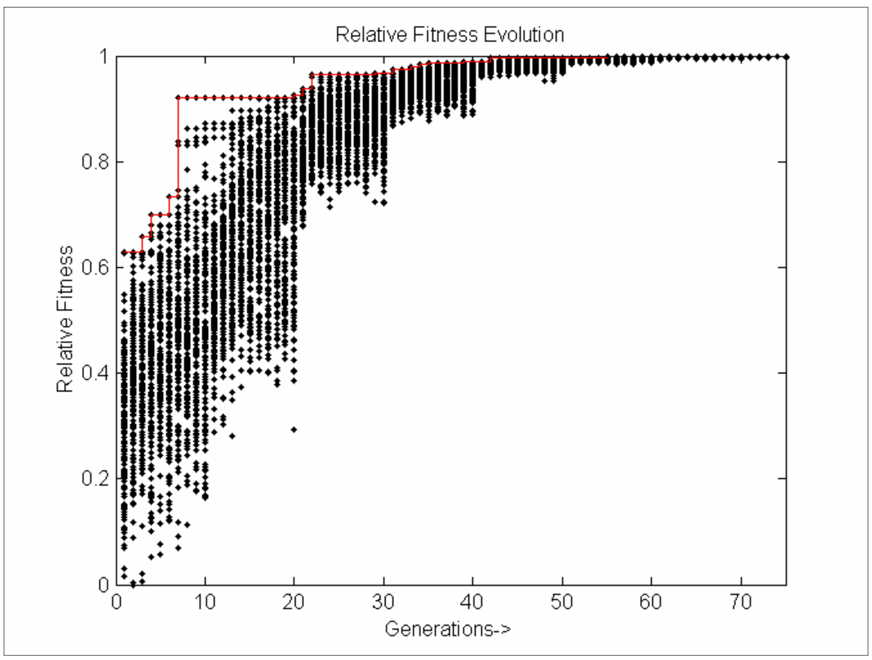

Figure 3. Evolution of the GA populations.

After every ten generations a restart population was randomly determined with a new parameter range. Figure 3 shows how after every ten generations the min and max errors fall within this reduced range. The algorithm user can allow this evolution to continue until their desired parameter precision is found.

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