Gravitational-Waveform Extraction by the Characteristic Method

Maria Babiuc-Hamilton
Department of Physics, Marshall University

Abstract

- When a pair of black holes spiral into each other and collide, the very fabric of space-time shakes, and gravitational waves are created.
- Gravitational waves carry information about their source, and will increase our understanding of relativistic systems in astrophysics.
- Gravitational wave observatories like LIGO and Virgo are tuned to detect the emission of these waves from the inspiral and merger of binary black holes, neutron stars, supernovae, etc...
- Problem: any small vibration is detected, so templates are essential to discern the real signal.
- It is hard to compute the waveforms obtained from numerical simulations accurately—gravitational radiation is properly defined only at null infinity, but is estimated at a finite radius.
- Cauchy-Characteristic Extraction (CCE) is the most precise and refined "extraction" method available. The CCE technique connects the strong-field “Cauchy” evolution of the space-time near the merger to the “characteristic" evolution far from the merger—at null infinity, where the waveform is extracted and detectors measure it.
- We present a stand-alone “characteristic" waveform extraction tool that has demonstrated accuracy and convergence of the numerical error and is used by the numerical relativity groups for the unambiguous extraction of waveforms.
- We prove that the numerical error of CCE satisfies the standards of the detection criteria required for Advanced LIGO data analysis.
- The tool provides a means for accurate calculation of waveforms generated by evolution codes based upon different analytic and numerical formulations of the Einstein equations.

Formalism

- Cauchy-characteristic method covers all space-time by combining 2 regions:
  1. A timelike (Cauchy) close to BBH
  2. A null (characteristic) far field.
- The characteristic evolution is embedded in the Cauchy evolution.
- Close quasicircular black hole binary inspiral with orbital frequency \( a_{\text{sg}} \).
- Characteristic initial data
  - Waveforms computed at null infinity in conformal compactified orbifold:
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Waves

- The waveform is extracted in terms of the Bondi News N and the Weyl tensor \( \Psi \):
  \[ N = N_0 + i N_1 = \alpha J_0 + \beta J_1 \]

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Advanced Accuracy Standards

- Sensitivity of detector, given in frequency domain
- Translated into the time domain, the error of a numerical waveform for strain and \( \Psi \) comp. \( \epsilon_{\Psi} \), \( \epsilon_{\psi_{i,j}} \), \( \epsilon_{\Psi_{ij}} \)
- Criteria for waveform accuracy
  1. Accuracy for extraction: \( \epsilon_{\Psi_{ij}} \)
  2. Accuracy for measurement: \( \epsilon_{\Psi_{ij}} \)
- Required accuracy or detection: \( \epsilon_{\Psi_{ij}} \), \( \epsilon_{\Psi_{ij}} \), \( \epsilon_{\Psi_{ij}} \)
- Required accuracy for measurement: \( \epsilon_{\Psi_{ij}} \), \( \epsilon_{\Psi_{ij}} \), \( \epsilon_{\Psi_{ij}} \)

Conclusions

- The aim of CCE is to provide a standardized gravitational waveform extraction tool.
- The new extraction tool contains major improvements and corrections to previous versions and displays convergence.
- The error introduced by CCE satisfies the time domain criteria required for advanced LIGO data analysis.
- The importance of accurate waveforms to the gravitational wave astronomy has created an urgency for tools like CCE.
- The source code has been released to the public and is available as part of the Einstein Toolkit.
- We welcome applications to a variety of generic numeric codes implementing Einstein Equations of General Relativity.

Requirements for the Bondi News

- The criterion for detection is satisfied throughout the entire binary mass range and is unaffected by choice of extraction radius.
- The criterion for measurement is also satisfied throughout the entire binary mass range.

Requirements for the Weyl Tensor

- The criterion for detection is satisfied throughout the entire binary mass range in the high mass limit.
- The values at all three extraction radii satisfy the measurement requirement for advanced LIGO signal-to-noise ratio \( p < 100 \).

Plots of the dominantly \((2, 2)\) mode of Richardson extrapolated waveform \( N^i (t) \) obtained with extraction radii \( R_s=20M, \, 50M, \) and \( 200M \).

- The \( R_s=20M \) and \( R_s=100M \) waveforms are shifted backward in time to be in phase at the peak.
- Two sources of “junk” radiation:
  1. Conflict of conformally flat initial Cauchy data
  2. Initial Cauchy and characteristic data mismatch
- The three waveforms are in good agreement in the inspiral and merger stage, with relative difference between the \( R_s=20M \) and \( R_s=100M \) to 0.04.

- The data is decomposed in Chebyshev and spherical harmonics coefficients on a band \( Rxdr \).
- Then is reconstructed in characteristic Bondi-Sachs coordinates, and evolved on the light cones.
- Cauchy solution with \( \beta \) is included in the computational grid by Penrose compactification of the radial coordinate:
  \[ \frac{dr}{d\tilde{r}} = e^\frac{\beta r}{2} \]
- Einstein equations \( g_{\mu\nu} \) evolved radially outward in Bondi-Sachs coordinates
  \[ d\tilde{r} = \left( 1 - \frac{\beta r}{2} \right)^{-1/2} dr + 2 \left( \frac{\beta r}{2} - 1 \right) \log\left( 1 + \frac{\beta r}{2} \right) d\log r \]
  \[ \Psi_{ij} = \left( 1 - \frac{\beta r}{2} \right)^{-1/2} \Psi_{ij} + 2 \left( \frac{\beta r}{2} - 1 \right) \log\left( 1 + \frac{\beta r}{2} \right) \Psi_{ij} + \left( 1 - \frac{\beta r}{2} \right)^{-1/2} \Psi_{ij} \]