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Testing a model for the well-posedness of the Cauchy-characteristic problem in Bondi coordinates

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Testing a model for the well-posedness of the Cauchy-characteristic problem

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What can we see with gravitational waves:
- Colliding black holes and galaxies,
- The birth of a black hole in a supernova
- The growth pains of our universe
- Gravitational waves are unambiguous measured only at future null infinity

Background

Babiuc and Winicour, Testing a model for the well-posedness of the Cauchy-characteristic problem
Cauchy-characteristic method covers all spacetime by combining 2 regions
1. A timelike (Cauchy) close to BBH
2. A null (characteristic) far field.

- Cauchy-characteristic initial-value
- Outward radial evolution
- Compactified radial coordinate
- Accurate gravitational radiation.

Formulation

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Is the null-timelike problem well-posed?

“As a general rule, it is considerably more difficult in the null case to write down formulae which say what one wants to say.”

R. Geroch, *Asymptotic Structure of Spacetime*


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**Question**

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Timelike & null initial boundary
Split the problem into:
- Cauchy problem
- Half-plane (strip) problem
Show that each individual problem is well posed.
Analyze stability against lower order perturbations.

Approach

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- **Real thing:** solve Einstein Equation of general relativity in Bondi-Sachs metric coordinates and calculate the gravitational waves.
- **Model:** solve the quasilinear wave equation in null-timelike compactified coordinates, on an asymptotically flat background with source, gived data on the timelike and initial boundary.

\[ g^{ab} \nabla_a \nabla_b \Phi = S(\Phi, \partial_c \Phi, x^c) \]

**Description**

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Change of variables: \( \Phi = e^{ax} \Psi, \ a > 0 \)

1+1 wave equation in characteristic coordinates:

\[
\begin{align*}
    t &= \tilde{t} - \tilde{x}, \ x = \tilde{t} + \tilde{x} \\
    \partial_t \partial_x \Phi &= S \rightarrow \partial_t (\partial_x + a) \Psi = F, \ F = e^{-ax} S, \ \Psi(0, x) = e^{-ax} f(x)
\end{align*}
\]

Energy estimates weighted norm is well-posed:

\[
E = \frac{1}{2} \int dx e^{-2ax} \left( (\partial_x \Phi)^2 + a^2 \Phi^2 \right)
\]
Discretization of the wave equation:

\[ \partial_t (D_{0x} U + a U) = LOT + S(t, x, y) \]
\[ U_0 = F(x, y), \quad (x, y) \in [0, 2\pi), \quad U(x, y) = U(x + 2\pi, y + 2\pi) \]

Discrete Fourier Transform:

\[ U(t, x, y) = \frac{1}{N} \sum_{0}^{N-1} \sum_{0}^{N-1} \hat{U}(t, \omega_1, \omega_2) e^{i(\omega_1 x + \omega_2 y)} \]
\[ \hat{U}(t, \omega_1, \omega_2) = \frac{1}{N} \sum_{0}^{N-1} \sum_{0}^{N-1} U(t, x, y) e^{-i(\omega_1 x + \omega_2 y)} \]

Equation to evolve:

\[ \partial_t \hat{U} = p \cdot \hat{U} + \hat{S} / d \]

Algorithm

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- When $\text{Re}(p)<0$, the solution decays exponentially fast
- When $\text{Re}(p)>0$, the solution grows exponentially fast
- When $\text{Im}(p)\neq 0$, the solution has oscillatory growing modes
- Even for $\text{Re}(p)<0$, time integration stability

\[
\text{Re}(p) = -\frac{a\omega_{21}^2 + 2b\omega_{10}\omega_{20} + \ldots}{a^2 + \omega_{10}^2}, \quad \text{Im}(p) = -\frac{-\omega_{10}\omega_{21}^2 + 2ab\omega_{20} + \ldots}{a^2 + \omega_{10}^2}
\]

Stability

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Start Variable

Evolved Variable

Pulse

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Goal: prove well-posedness of quasilinear equation in the null-characteristic domain

- Prove well-posedness of Cauchy problem
- Study the effect of lower order perturbations
- Apply boundary on the initial characteristics
- Restrict the problem to timelike-null domain
- Extend the analysis to the quasilinear wave

Checklist

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Thank You

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